John Napier (1550–1617)

The late sixteenth century was an age of numerical computation, as developments in astronomy and navigation called for increasingly accurate and lengthy trigonometric computations. Georg Joachim Rheticus (1514–1576) began the computation of a great collection of 15-place trigonometric tables which were completed and published by Otho in 1596 and by Pitiscus in 1613. The urgent need, for some device to shorten the labor of tedious multiplications and divisions with many decimal places, was met through the invention of logarithms by Napier and others around the turn of the seventeenth century.

John Napier was the eighth baron (or laird) of Merchiston. He is said to have regarded his book *A Plaine Discovery of the Whole Revelation of Saint John* (1593) as his most important contribution. This polemical tract contained proofs in Euclidean fashion that the Pope was the Antichrist and that the world was due to end in the year 1786. With this theological work behind him, he began in 1594 the work that was to revolutionize the practical art of numerical computation. This labor occupied a twenty-year period spent in the isolation of Merchiston castle near Edinburgh in the south of Scotland.

Napier's logarithmic tables first appeared in 1614 in a small book entitled *Mirifici Logarithmorum Canonis Descriptio* (Description of the Wonderful Canon of Logarithms), which contained only an introduction and guide to the computational use of the tables. The method of computation of the tables themselves, and to a lesser extent the reasoning upon which they were based, were summarized in the *Mirifici Logarithmorum Canonis Constructio* (Construction of the Wonderful Canon of Loga-
rithms), the first written of the two books, but published posthumously in 1619. Extracts from an 1889 English translation of the *Constructio* by W. R. Macdonald may be found in the Napier tercentenary memorial volume ([NT], pp. 25–32) or in D. J. Struik's mathematics source book ([11], pp. 11–21).

The practical advantages, of using logarithms to convert tedious multiplications and divisions to comparatively simple additions and subtractions, were immediately obvious. For example, when Kepler received Napier's tables of 1614, he enthusiastically employed them in the enormous computations that led to the discovery of his third law of planetary motion.

Today we think of the logarithm \( \log_a x \) of the number \( x \) (with base \( a \)) as the power to which \( a \) must be raised to obtain \( x \). However, in order to properly gauge the magnitude of Napier's accomplishment, it is important to realize that fractional powers and exponential notation had in Napier's time not yet been developed. Neither was the decimal point system of numeration generally accepted. Indeed, it was Napier's systematic use of decimal points that was largely responsible for the general adoption of decimal point notation during the seventeenth century.

In particular, we think of the logarithm as a function, or even as the inverse of an exponential function. However, Napier's computations were based on a clear understanding of a particular functional relationship at a time when the general concept of a function was still unknown. Indeed, the logarithm function played a prototype role in the development of this general concept. Also, as we shall see in this chapter, the study of logarithms led to the calculation of hyperbolic areas (such as the area under the rectangular hyperbola \( xy = 1 \)). In these ways the logarithm function, in addition to its computational importance, played a significant role in the historical development of the calculus.

The Original Motivation

The object of Napier's "wonderful canon of logarithms" was to reduce the tedious operation of multiplication to the much simpler operation of addition by means of the correspondence between an arithmetic series and a geometric series. In his *Arithmetica Integra* of 1544, Michael Stifel (with whose work Napier is likely to have been familiar) set down side-by-side the arithmetic and geometric series

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \cdots \\
1 & 2 & 4 & 8 & 16 & 32 & 64 & 128 & 256 & \cdots
\end{array}
\]

and pointed out that addition in the upper (arithmetic) series corresponds to multiplication in the lower (geometric) series (see pp. 85–86 of D. E.