Suppose we are studying a physical system whose state $x$ is governed by an evolution equation $\frac{dx}{dt} = X(x)$ which has unique integral curves. Let $x_0$ be a fixed point of the flow of $X$; i.e., $X(x_0) = 0$. Imagine that we perform an experiment upon the system at time $t = 0$ and conclude that it is then in state $x_0$. Are we justified in predicting that the system will remain at $x_0$ for all future time? The mathematical answer to this question is obviously yes, but unfortunately it is probably not the question we really wished to ask. Experiments in real life seldom yield exact answers to our idealized models, so in most cases we will have to ask whether the system will remain near $x_0$ if it started near $x_0$. The answer to the revised question is not always yes, but even so, by examining the evolution equation at hand more minutely, one can sometimes make predictions about the future behavior of a system starting near $x_0$. A trivial example will illustrate some of the problems involved. Consider the following two
differential equations on the real line:

\[ x'(t) = -x(t) \quad (1.1) \]
and

\[ x'(t) = x(t) \quad (1.2) \]

The solutions are respectively:

\[ x(x_0, t) = x_0 e^{-t} \quad (1.1') \]
and

\[ x(x_0, t) = x_0 e^{+t}. \quad (1.2') \]

Note that 0 is a fixed point of both flows. In the first case, for all \( x_0 \in \mathbb{R}, \lim_{t \to \infty} x(x_0, t) = 0. \) The whole real line moves toward the origin, and the prediction that if \( x_0 \) is near 0, then \( x(x_0, t) \) is near 0 is obviously justified.

On the other hand, suppose we are observing a system whose state \( x \) is governed by (1.2). An experiment telling us that at time \( t = 0, x'(0) \) is approximately zero will certainly not permit us to conclude that \( x(t) \) stays near the origin for all time, since all points except 0 move rapidly away from 0. Furthermore, our experiment is unlikely to allow us to make an accurate prediction about \( x(t) \) because if \( x(0) < 0, x(t) \) moves rapidly away from the origin toward \( -\infty \) but if \( x(0) > 0, x(t) \) moves toward \( +\infty \). Thus, an observer watching such a system would expect sometimes to observe \( x(t) \xrightarrow{t \to \infty} -\infty \) and sometimes \( x(t) \xrightarrow{t \to \infty} +\infty \). The solution \( x(t) = 0 \) for all \( t \) would probably never be observed to occur because a slight perturbation of the system would destroy this solution. This sort of behavior is frequently observed in nature. It is not due to any nonuniqueness in the solution to the differential equation involved, but to the instability of that solution under small perturbations in initial data.