GENERALIZED DUAL SPACE INDICATOR METHOD FOR IMAGING OBSTACLES IN A HOMOGENEOUS MEDIUM

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Abstract

This article presents a generalized dual space indicator method for imaging an obstacle in a homogeneous medium. The method is based on the observation that the combination (weighted integration) of the measured scattered field can approximate Green's function very well when the Green's function's source point is inside of the obstacle, but not so well when the source is outside of the obstacle. We set up an integral equation whose right-hand-side is the Green's function with source point from a searching region. The solution of the integral equation has noticeable different norms for the source being in or out of the obstacle. Plotting the norm as a function of the source point in the searching region, we obtain
Introduction

In [4], a generalized dual space indicator method is developed for imaging an obstacle in ocean environments. This method is inspired by a recent method of Colton and Kirsch [2] using far-field pattern in a homogeneous space, and by the matched-field signal processing method for source localization (see for example [5], [3]). The method is based on the observation that the combination (weighted integration) of the measured scattered field can approximate Green’s function very well when the Green’s function’s source point is inside of the obstacle, but not so well when the source is outside of the obstacle. We set up an integral equation whose right-hand-side is the Green’s function with source point from a searching region. The solution of the integral equation has noticeable different norms for the source being in or out of the obstacle. Plotting the norm as a function of the source point in the searching region, we obtain a good image of the unknown obstacle. The most remarkable advantage of this method is that it does not require any a priori knowledge of what kind of boundary condition is imposed on the obstacle.

In this paper we consider as our model the case of scattering of acoustic waves in a homogeneous deep ocean by an infinite cylinder with bounded cross section \( \Omega \). We consider the incident acoustic wave is from a line source parallel to the axis of the cylinder. We assume that the cylinder is wave inpenetrable. We make little assumptions on the kind of boundary conditions the acoustic wave satisfies. The total acoustic wave could satisfy any one of the Dirichlet, Neumann or impedance condition. Under the above assumptions, the total acoustic field \( u \) satisfies a 2-dimensional Helmholtz equation

\[
\Delta u(x) + k^2 u(x) = \delta(x - x^o), \text{ for } x \in \mathbb{R}^2 \setminus \Omega
\]  

(1)

\[
\lim_{r \to \infty} \sqrt{r} \left( \frac{\partial u}{\partial r} - iku \right) = 0
\]  

(2)

where \( k > 0 \) is the wavenumber, \( r = |x| \) and \( x^o \) is the location of the acoustic source. \((x^o \) is the cross section of the line source, i.e., a point in the \( \mathbb{R}^2 \) plane). We consider the total wave \( u \) as the combination of a