VARIATIONAL PRINCIPLE FOR CRITICAL HEAT OF QUENCH IN PARTIALLY STABILISED SUPERCONDUCTING MAGNETS

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ABSTRACT

The theory based on the minimum propagating zone (MPZ) criterion overestimates the critical heat of quench in the partially stabilized superconducting magnets. This difference is especially important for the high values of Stekly parameter. We discuss the variational principle for calculation of the critical heat of quench and compare our results with that of based on the MPZ theory and obtained by the direct dynamic numerical simulations. It is shown that variational calculations can effectively be used for the determination of the critical heat. At the end, we discuss the Lyapunov approach to the stability problem.

INTRODUCTION

The superconducting magnets with partial cryogenic stabilization remain now the most widespread devices used in various applications. They are thermally bistable systems. The superconducting state will be recovered after the action of relatively weak thermal disturbances, while the strong disturbances will cause the unbounded growth of resistive zone and the quench of a magnet. The global stability characteristics by Stekly and Zar' have then been extended by Maddock et al.\(^2\) to the local disturbances. It has been suggested\(^3\) that the enthalpy corresponding to the stationary spatially inhomogeneous solution of the heat balance equation (called the minimum propagating zone\(^3\) (MPZ)) should be considered as the minimum critical energy which may cause a quench. The extended investigations based on this criterion has been considered by many authors (see, e.g.,\(^4,5,6,7\) and references therein). However, it's now well understood that original MPZ theory is applicable only for the rather narrow class of thermal disturbances and overestimates generally the critical heat. The difference is especially important for the magnets operating in relatively bad cooling conditions (for the high values of Stekly parameter\(^5,6\)). One of the alternatives to the MPZ theory consist in the direct dynamic simulations for the wide class of initial temperature profiles. We will show how the original MPZ theory can be modified in or-
der to assess more realistic estimate for minimum critical heat of quench. The advantage of our method consists in the variational solution of some effective static heat flow equation rather than the dynamic one. The first problem is naturally much simpler. At the end, we will briefly discuss the alternative approach to the stability problem based on the Lyapunov theory and further generalizations.

VARIATIONAL PRINCIPLE

We first describe the general scheme. The similar ideas has also been discussed by Wilson. The effective heat flow equation can be written as

\[ C(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial r} \left( \gamma_{\alpha\beta} \frac{\partial T}{\partial r} \right) + G(T) - Q(T). \]  

where \( C(T) \) is the specific heat per unit volume, \( \gamma(T) \) is thermal conductivity, \( G(T) \) and \( Q(T) \) are the effective heat generation and cooling per unit volume. The tensor \( \gamma_{\alpha\beta} \) describes the anisotropy effects. Here the indices \( \alpha, \beta \) correspond to Cartesian or cylindrical coordinates and summation is performed over repeating indices in Eq.1. The winding is also cooled at the boundary:

\[ - \gamma(T) n_\alpha \gamma_{\alpha\beta} \frac{\partial T}{\partial r} \bigg|_B = Q_A(T). \]  

where \( n \) is the normal to the winding surface directed outside and \( Q_A(T) \) is the cooling per unit area. In the problem concerned it can be taken that \( G(T_B) = Q(T_B) = Q_A(T_B) = 0 \), where \( T \) is bath temperature. Thus, there is stationary spatially homogeneous solution of Eqs.(1) and (2) \( T(r) = T_B \) which corresponds to the initial operating regime. The system is assumed to be bistable and there is also the MPZ-like stationary solution, \( T_{MPZ}(r) \). Let us now consider a system with the additional coolings instead of Eqs.(1) and (2):

\[ \tilde{Q}(T,r) = Q(T) + \Delta Q(T,r). \]  

\[ \tilde{Q}_A(T,r) = Q_A(T) + \Delta Q_A(T,r). \]  

where the functions \( \Delta Q(T,r) \) and \( \Delta Q_A(T,r) \) are non-negative everywhere and are equal to zero if \( T \leq T_b \). Let \( T_{MPZ}^{\sim}(r) \) be the stationary MPZ profile in a system with additional cooling. Then such a profile will evolve to the normal state in the initial system with the cooling \( Q(T) \) and \( Q_A(T) \), since it is a separatrix profile in a system with the better cooling conditions. Let us define the minimum critical energy of quench as

\[ E_{cr} = \min \{ E_{MPZ} \} = \min \left\{ \int_{T_{MPZ}^{\sim}}^{T_{cr}^{\sim}} dT \right\} \equiv \int_{T_b}^{T_{cr}} dT \int dT \left( C(T) \frac{\partial T}{\partial t} + \frac{\partial}{\partial r} \left( \gamma_{\alpha\beta} \frac{\partial T}{\partial r} \right) + G(T) - Q(T) \right). \]  

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