BACKSCATTER FROM A SPHERICAL INCLUSION WITH COMPLIANT INTERPHASE CHARACTERISTICS

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INTRODUCTION

In studies of scattering by an inclusion it is generally assumed that the inclusion is perfectly bonded to the surrounding matrix material. The actual bond between two materials is, however, generally effected by a thin layer, which may be called an interphase, rather than an interface. It is well known that the mechanical behavior of such an interphase may significantly influence the overall mechanical behavior of a solid containing inclusions.

In this paper it is investigated to what extent an interphase affects the scattered field generated by an incident ultrasonic wave. Both a completely intact but compliant interphase, and an interphase which does not transmit tractions over part of the area between the inclusion and the matrix, have been considered.

The interphase is generally very thin. In this paper, it is assumed that the radial and the tangential tractions are continuous across the interphase, but the displacements may be discontinuous from inclusion to matrix. The tractions are assumed to be proportional to the corresponding displacement discontinuities. The proportionality constants characterize the stiffness and strength of the interphase. On the basis of this interphase model, which corresponds to a distribution of springs between the inclusion and the matrix, a rigorous analysis has been carried out of the backscattered field generated by an incident longitudinal wave. Within the context of the present model the case of a partially defective interphase, i.e., the case of a crack over part of the surface between the inclusion and the matrix, is easily included by setting the spring constants identically zero over the cracked surface.

The analysis has been carried out by deriving a set of singular integral equations for the tractions and displacements across the
interphase. These equations have been solved by the boundary element method, and the scattered field has subsequently been obtained by the use of the elastodynamic integral representation.

An analysis for a similar configuration, but by the use of the null field approach, and for a completely intact interphase, has been presented by Datta, Olsson and Boström [1].

FORMULATION

Let \( \lambda \) and \( \mu \) be Lamé's elastic constants, and \( \rho \) the mass density of the inclusion, and let \( \lambda, \mu, \rho \) be the corresponding quantities of the surrounding matrix material, as shown in Fig. 1. In this figure, \( u^i \) and \( u^s \) are the incident and the scattered wave, respectively. The total wave field is defined as \( u = u^i + u^s \). In the following, the time-harmonic factor \( \exp(-i\omega t) \) has been suppressed, and the upper bar notation is used for quantities related to the inclusion.

Boundary Integral Equations for the Matrix and the Inclusion

The boundary integral equation for the matrix material takes the form

\[
C_{ij}\left( x \right) u_j\left( x \right) = \int_S U_{ij}(x,y)T_j(y)\,dS_y - \int_S T_{ij}(x,y)u_j(y)\,dS_y + u_i^I(x), \quad x \in S,
\]

where \( S \) is the interphase boundary at the matrix side and \( U_{ij}(x,y) \) is the fundamental solution for 3D time-harmonic elastodynamics:

\[
U_{ij}(x,y) = \frac{1}{4\pi\mu} \left[ \frac{ikr}{r} \delta_{ij} + \frac{i}{k} \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} \left( \frac{e^{ikr}}{r} - e^{ikLr} \right) \right],
\]

while \( T_{ij}(x,y) \) is the corresponding traction. The boundary integral equation for the inclusion is of the form

\[
\bar{C}_{ij}(x)\bar{u}_j(x) = \int_{\bar{S}} \bar{U}_{ij}(x,y)\bar{T}_j(y)\,dS_y - \int_{\bar{S}} \bar{T}_{ij}(x,y)\bar{u}_j(y)\,dS_y, \quad x \in \bar{S},
\]

Fig. 1 Scattering by an inclusion with a compliant interphase.