ON THE COVARIANT QUANTIZATION OF ANOMALOUS GAUGE THEORIES

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The standard model of gauge theory for weak and electromagnetic interactions describes fermions as having a definite chirality: left and right chiralities are so different that for example the left handed part of the electron and its right-handed part (each of which is a Weyl fermion) do not have the same quantum numbers (weak hypercharge for example) [1,2], and thus it is natural to try to quantize gauge theories with Weyl fermions.

Unfortunately the presence of Weyl fermions in a gauge theory is known to produce, at the quantum level, non-gauge-invariant interactions. This is the phenomenon of anomalies [3].

The presence of anomalies in the theory is a disease which one usually avoids by carefully choosing the fermion content, to force mutual cancellations between the various non-invariances possibly produced by each fermion [4,5].

Another cancellation mechanism is to add extra fields in the theory, e.g. a group valued field with definite transformation as introduced by Wess and Zumino [6], or other fields coming from a larger theory and a judicious choice of the group of symmetry [7].

Our point, described in a work of O. Babelon, F. Schaposnik and the author [8] (see also the work of K. Harada and T. Tsutsui [9]) is to show that the use of lagrangian formalism and functional integral formalism leads, in the case of anomalous gauge theories -with no forced cancellations- to reconsider the quantization rules [10].

Our motivation is ...

... 1) The idea promoted by Faddeev and Shatashvili [11,12] in the context of hamiltonian formalism, together with the need of explicit Lorentz invariance.

... 2) The results obtained by Polyakov in a similar problem for the conformal anomaly in the context of string theory [13].

We thus adopt the lagrangian formalism and use the functional integral

\[ Z = \int DA \: \theta \gamma \: e^{-S}. \]
with \( S = \int \mathcal{L} \, \text{d}^4x \), \( \mathcal{L} = \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{YM}} = \overline{\psi}_L \left( i \gamma^\mu \partial_\mu + A_\mu \right) \psi_L + \frac{1}{4} \text{tr} F_{\mu\nu}^2 \).

where \( \psi_L \) is a left-handed Weyl fermion: \( \psi_L = \frac{i}{2} \gamma^\mu \psi_L \).

The outcome is that if one treats properly the functional integration, then the group valued field of Wess-Zumino is not an external field and it is present in the theory. Actually the Wess-Zumino action emerges and one recovers independence on the choice of gauge condition.

In other words, the gauge part of the gauge potential, which is irrelevant at the classical level [14] acquires a different status in the quantum theory (Exactly the same phenomenon happens for two dimensional metrics in [13]). The core of the discussion is the correct account of the number of degrees of freedom of the quantum theory, which may differ from the one for the classical theory.

1. We first recall some results [15] on pure gauge theory, coming from the geometrical analysis of the Faddeev-Popov determinant [10].

2. We then recall results [16] on the fermionic measure in the case where Weyl fermions are present.

3. With this in hand we reconsider the full functional integral, including integration over gauge potentials and integration over the fermionic degrees of freedom, without changing the starting point - we pretend to study Yang-Mills theory - but merely keeping in mind all features of the objects we write down. In the absence of anomalies we recover the standard results.

4. We give some explicit formulae for the 2-dimensional case.

I. THE GEOMETRICAL CONTENT OF THE USUAL QUANTIZATION RULE OF GAUGE THEORY

In usual Yang-Mills theory, in the lagrangian approach, one writes a functional integral over the space of gauge potentials (pure Yang-Mills)

\[ Z_{\text{YM}} = \int \mathcal{D}A \, e^{-S_{\text{YM}}} \]

This integral is a priori an integral over the whole space \( \mathcal{C} \) of gauge potentials

The invariance of \( S_{\text{YM}} = \frac{1}{4} \int \text{tr} F_{\mu\nu}^2 \) under gauge transformations means that \( S_{\text{YM}}(A) \) is constant on any orbit of the group \( \mathcal{G} \) of gauge transformations. This invariance is a trivial source of divergence for the integral \( Z_{\text{YM}} \) and leads to choosing a gauge section \( \mathcal{A} \) and restricting the integral to \( \mathcal{A} \). Such a choice of \( \mathcal{A} \) is the choice of a unique representative in each orbit.

The choice of \( \mathcal{A} \) may be done by imposing a differential equation on the gauge potential, eg \( \mathcal{A} = 0 \). The independence on the choice of \( \mathcal{A} \) persists only through the presence of an additional term in the measure: