A BRIEF INTRODUCTION TO SPIN GLASSES
AND RELATED COMPLEX PROBLEMS

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ABSTRACT

I give a brief semihistorical review of progress in understanding spin glasses, with emphasis on the novel features of mean field theory and applications to other "complex systems": optimization problems and neural networks.

I. INTRODUCTION

In this lecture I will try to summarize what I think are the most important aspects of spin glass theory. Because of the limited time, the treatment will have to be rather sketchy and descriptive, without analytic derivation of most results. The choice of topics is also biased by my opinion that what is most important about spin glasses is that they are a prototype of a new class of problems in statistical physics. These problems are qualitatively more complex than those we are used to dealing with in condensed matter physics and require new theoretical tools and concepts. A number of people nowadays hope that their solution will help point the way to significant advances in understanding the kinds of complex collective behaviour we see in, for example, biological or economic systems.

What, then, is a spin glass? On the basis of accumulated empirical knowledge, we can say the following: It is a disordered system of interacting spins (or spin-like degrees of freedom) in which there is competition between the interactions. As a consequence of these features, no single configuration or small number of configurations of the spins is uniquely favorable energetically - a spin glass can exist in many possible states.

So we do not have the kind of situation we find in ordered ferromagnets and other conventional broken-symmetry systems, where the kind of order is relatively simple to
understand and to characterize mathematically. In a spin glass, the way one characterizes the order is intimately tied up with the fact that there is randomness in the system and that there are many possible states. This raises very general questions about the nature of possible "order amid disorder", one fundamental reason why spin glasses have been viewed as a problem of fundamental interest.

This feature, together with the hope I mentioned above - that understanding spin glasses could be a key that unlocks the secrets of many other complex systems in and out of physics - have made this field one of the most active ones in theoretical physics in the last decade. I will now try to summarize some of the progress that has been achieved in understanding physical spin glasses and two of the other kinds of problems - combinatorial optimization problems and so-called neural networks.

II. SPIN GLASSES

One can identify a number of important experimental signatures of spin glasses. The most important of these is the well-known cusp in the susceptibility at $T_g$, the spin glass transition temperature. Together with the knowledge (from neutron scattering) that there is no long-range periodic magnetic order, this gives direct evidence of the spins freezing into an irregular, "glassy" configuration.

On closer examination, $T_g$ turns out not to be completely sharply defined. The peak is slightly rounded and slightly frequency-dependent. Furthermore, at $T<T_g(\omega)$, $\chi$ also shows $\omega$-dependence. The characteristic times in the system evident from this $\omega$-dependence stretch from microscopic times ($10^{-12}$ sec) up to the longest times practically accessible in an experiment ($10^6$ sec), without any noticeable gap. This is in sharp contrast to ferromagnets and other conventional ordered magnets, where a gap of many orders of magnitude separates microscopic characteristic times from the single macroscopically long (usually unobservably so) typical time between overall flips of the net order parameter. This extremely broad spectrum of characteristic times strongly suggests that spin glasses have very many locally stable configurations (unlike conventional magnets), separated by energy barriers of varying height.

A third principal signature of a spin glass is remanence and irreversibility below $T_g$. The most dramatic example of this is the difference between the so-called field-cooled susceptibility (defined by turning on the magnetic field above $T_g$, cooling in the field and then measuring the induced magnetization) and the zero-field-cooled $\chi$ (where the field is turned on after the system has been cooled). The former is roughly temperature-independent, while the latter falls as $T$ is lowered below $T_g$. These effects, too, are a consequence of the existence of many possible metastable states.

All these phenomena are remarkably universal, differing only in numerical detail from one system to another or from one class of systems to another, superficially different, class. Although they have been most thoroughly studied in the classic spin glass systems, the RKKY glasses, consisting of dilute transition metal magnetic impurities in noble metal hosts, they also occur in insulating magetic alloys, amorphous magnets, and mixed ferroand antiferroelectric crystals, where the "spin" is really an electric rather than a magnetic moment. In still other systems, the "spin" seems effectively to be an electric quadrupole moment. While some details of the phenomena may not be completely universal, or there may be universality classes and subclasses that have not been fully sorted out yet, the qualitative aspects are. This fact suggests that the best strategy for trying to understand these

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