In this article I shall briefly review the currently proposed theoretical models for the discovered high transition temperature superconductivity in copper oxides in connection with the existing experimental results.

The discovery of high transition temperature superconductors in Cu-oxide compounds by Bednorz and Müller, and the subsequent findings of $T_C$ above the liquid nitrogen point have made revolutionary changes in the field of superconductivity. Competitions all over the world on raising $T_C$ higher are still going on. In the past several months, hundreds of experimental groups have reported various properties of these superconducting materials. In addition to the extra high $T_C$, these compounds behave quite differently from usual ones in many aspects. The discovery came as a big surprise to all the theorists. Although many efforts had been made to the problem of raising $T_C$, the metal oxide compounds were out of the expectation. The revolution has greatly stimulated the theorists in condensed matter physics. From different viewpoints, a number of theoretical models have been proposed to explain the observed high $T_C$ superconductivity. Because $T_C$ is so high, a central question is what is so special about Cu-oxides. In this article I shall briefly review the proposed theoretical models in connection with the existing experimental results. It is perhaps fair to say that theorists have not yet understood these new materials. We are searching for the truth. The explanation of the high $T_C$ superconductivity mechanism is, of course, the most important task in many-body theory at present.

All theories so far have concentrated on $La_{2-x}M_xCuO_4$, with $M$ being Sr or Ba. The Y-based compounds have higher $T_C$, but are electronically similar although more complicated in crystal structure. Experimentally it is more flexible to change the doping concentrations in the La-based compounds. It is generally assumed the underlying mechanism essentially to be the same for the Y-compounds.

Since the pure $La_2CuO_4$ is a semiconductor or an insulator at low temperatures, replacing small amount of La by Sr changes the system to superconductors, any theory must explain an insulating gap for the pure system,
and examine the role of doping for the superconducting states. The undoped superconductor $\text{La}_2\text{CuO}_4$ can be explained due to the excess of oxygen, an effect equivalent to the doping from the theoretical point of view for most of the models. There are two types of ideas on the mechanisms, the conventional BCS electron-phonon theory and the new superconducting mechanisms.

To start with, we shall consider the band structure picture. The valence of Cu atom is 2+ in the reference compound $\text{La}_2\text{CuO}_4$ if we adopt a simple chemical assignment for the La-atoms 3+ and O-atoms 2-. Cu 3d electrons with eg symmetry strongly hybridize with 0 2p electrons. Because of the large elongation of the O-octahedra perpendicular to the Cu-O planes, the eg orbital degeneracy is further removed, leading to a single band with one electron per Cu site. The spin of the Cu atom is 1/2 due to the strong crystal field. The interplane coupling and the next nearest neighbor coupling of Cu atoms in the plane are expected to be quite small. This leads to a tight binding Hamiltonian in a square lattice

$$H_0 = -t \sum_{\langle ij \rangle} C_i^+ C_j^0 + \text{h.c.}, \quad (1)$$

where $t$ is the hopping integral between the nearest neighbor Cu atoms $i$ and $j$. The detailed band structure calculations of Mattheiss and Yu, Freeman and Xu have clearly suggested such a picture.

The non-interacting Hamiltonian of Eq. (1) has a simple dispersion relation

$$\epsilon(k) = -2t(\cos k_x a + \cos k_y a) \quad . \quad (2)$$

The Fermi surface at half-filled case is perfectly nesting in the (1,1) orientation, and the density of states at the Fermi surface is logarithmic divergent due to the saddle points. Without interaction, the system would be metallic. But such a Fermi surface is strongly unstable against the lattice distortion or spin density wave (SDW). The important interactions of the system are the electron-phonon coupling $H_{\text{e-ph}}$ and the on-site Coulomb repulsive interaction $U$. Thus we may describe the Cu-oxide planes by

$$H = H_0 + H_{\text{e-ph}} + U \sum_i n_{i\uparrow} n_{i\downarrow}. \quad (3)$$

The last term is called Hubbard term, and $n_{i\sigma} = C_{i\sigma}^+ C_{i\sigma}$. The interplane coupling is weak, but is assumed sufficient to produce the superconducting phase transitions in the planes.

The physics of Eq. (3) depends on the parameters. If electron-phonon coupling is dominant, then the effective on-site interaction $U_{\text{eff}}$ is attractive, and the system at 1/2-filled is unstable against some kinds of lattice distortions. Otherwise $U_{\text{eff}}$ is repulsive, and SDW is expected. If $U$ is very large in comparison with $t$, one has a Mott-Hubbard insulator. The superconductivity has been theoretically proposed in the all parameter regions for both weak and strong coupling limits.

A common viewpoint of the weak coupling theory for the high $T_c$ materials is that near the lattice or SDW instability, the superconducting $T_c$ could be enhanced. In $\text{La}_2\text{CuO}_4$, the band is exactly half-filled. Doping of Sr removes the perfect nested Fermi surface, hence suppressing the commensurate charge density wave (CDW) or SDW transition with the wave number.