THREE LECTURES ON FLAVOUR MIXING

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1. INTRODUCTION

The observation by ARGUS$^1$ at DESY of a relatively large amount of $B^0$-$\bar{B}^0$ mixing, following a previous positive signal of mixing by UA1$^2$, was the most important experimental result of the year in particle physics (together with the very recent result on $e^+/e^-$ by the NA3$^3$ collaboration at CERN). The UA1 result was already known last year. The ARGUS result refers to the $B^0_d$ meson ($B^0_d \equiv b\bar{d}$, $B^0_s \equiv b\bar{s}$). In terms of $r = P(B^0 \rightarrow B^0)/P(\bar{B}^0 \rightarrow \bar{B}^0)$, i.e., the ratio of the probability for mixing and for no mixing, ARGUS finds:

$$r = 0.21 \pm 0.08$$  \hspace{1cm} (1.1)

The experimental method was described in the lectures by S.L. Wu$^4$. On the theoretical side a large number of papers have been devoted on $B^0$-$\bar{B}^0$ mixing in the past$^5$ and then recently$^6$-$^10$ after the UA1 and ARGUS results. These lectures are intended to an elementary introduction to flavour mixing in general and to $B^0$-$\bar{B}^0$ mixing in particular. Their purpose is to provide the reader with the essential background necessary to follow the current specialized literature.

2. BASIC FORMALISM

For a stable free particle at rest the quantum mechanics time evolution is given by $\psi \sim e^{-iMt}$. For an unstable particle at rest, this is modified into $\psi \sim e^{-i(M-i\Gamma/2)t}$ (in fact, $|\psi|^2 \sim e^{-\Gamma t} = e^{-t/\tau}$), with $M$ and $\Gamma$ real, positive numbers. For several coupled states $M$ and $\Gamma$ became Hermitian matrices with positive eigenvalues (i.e., the analogue of real, positive numbers). For several coupled states $M$ and $\Gamma$ became Hermitian matrices with positive eigenvalues (i.e., the analogue of real, positive numbers). In particular for $B^0$-$\bar{B}^0$ (or any other similar system), we have:

$$^\prime\prime H^\prime\prime \left( \frac{B^0}{\bar{B}^0} \right) = \begin{pmatrix}
M - i \Gamma_2/2 & M_{12} - i \Gamma_{12}/2 \\
M_{12}^* - i \Gamma_{12}/2 & M - i \Gamma_2/2
\end{pmatrix} \begin{pmatrix}
\frac{B^0}{\bar{B}^0}
\end{pmatrix}$$  \hspace{1cm} (2.1)

Note that a) $^\prime\prime H^\prime\prime$ is not Hermitian (since probability is not conserved within the $B^0$-$\bar{B}^0$ system, because of the decays); b) $H_{11} = H_{22}$ by CPT;
c) $H_{12} \neq 0$, $H_{21} \neq 0$ because of the weak interactions which violate the conservation of quark flavours. d) $\text{Im} M_{12} \neq 0$, $\text{Im} \Gamma_{12} \neq 0$ because of CP violation.

The eigenvalues of $"H"$ can be written down in the form:

$$B_{1,2} = \frac{(1+\varepsilon) B_0 \pm (1-\varepsilon) B_0}{\sqrt{2(1+|\varepsilon|^2)}}$$

Note that $B_1$ and $B_2$ are not orthogonal because $"H"$ is not Hermitian. If $\varepsilon = 0$, CP is conserved in the wave functions. In general, $\varepsilon$ depends on the phase convention chosen. Thus, for example, $\varepsilon$ pure imaginary does not lead to any CP violation because it can be removed by a redefinition of the relative $\Upsilon^0-\Upsilon^0$ phase. A simple calculation immediately leads to the following results for $\varepsilon$ and the eigenvalues of $M$ and $\Gamma$:

$$\eta = \frac{1-\varepsilon}{1+\varepsilon} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}/2}{M_{12}^* - i\Gamma_{12}/2}}$$

$$M_{12} = M \pm \text{Re} Q$$

$$\Gamma_{12} = \Gamma \mp 2 \text{Im} Q$$

where $M$, $\Gamma$, $M_{12}$ and $\Gamma_{12}$ are defined in Eq. (2.1) and

$$Q = \sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}/2)}$$

$\Upsilon^0-\Upsilon^0$ oscillations are caused by the different evolution in time of the eigenvectors $B_1$ and $B_2$. Starting at $t = 0$ from a pure $\Upsilon^0$ state:

$$|\Psi(t=0)\rangle = |B^0\rangle = (|B_1\rangle + |B_2\rangle) \frac{\sqrt{1+|\varepsilon|^2}}{\sqrt{2(1+|\varepsilon|^2)}}$$

one obtains at time $t$:

$$|\Psi(t)\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2(1+|\varepsilon|^2)}} \left[ |B_1\rangle e^{-i(M_1 - \frac{1}{2}\Gamma_1)t} + |B_2\rangle e^{-i(M_2 - \frac{1}{2}\Gamma_2)t} \right]$$

By using Eq. (2.2) we can eliminate $|B_1\rangle$ and $|B_2\rangle$ and write $|\Phi(t)\rangle$ as a superposition of $\Upsilon^0$ and $\Upsilon^0$. The coefficients are the transition amplitudes: $A(B\rightarrow B)$ and $A(B\rightarrow \overline{B})$. One immediately obtains:

$$A(B\rightarrow B) = \frac{1}{2} \left[ e^{-iM_1 t} e^{-\frac{1}{2}\Gamma_1 t} + e^{-iM_2 t} e^{-\frac{1}{2}\Gamma_2 t} \right]$$

$$A(B\rightarrow \overline{B}) = \frac{1-\varepsilon}{1+\varepsilon} \frac{1}{2} \left[ e^{-iM_1 t} e^{-\frac{1}{2}\Gamma_1 t} - e^{-iM_2 t} e^{-\frac{1}{2}\Gamma_2 t} \right]$$

We define the ratio $r$ of total (i.e., integrated over time) probabilities:

$$r = \frac{P(B\rightarrow \overline{B})}{P(B\rightarrow B)} = \frac{\int_{0}^{T} |A(B\rightarrow \overline{B})|^2 \, dt}{\int_{0}^{T} |A(B\rightarrow B)|^2 \, dt}$$

where $T$ is a conveniently large time. One directly obtains from Eqs. (2.9) and (2.10):