Chapter 2

Discrete–Event processes

A stochastic process (indexed with time) is a collection of random variables $(Z(t,\omega))_{t\in\mathbb{R}^+}$ defined on some probability space $(\Omega, \mathcal{A}, \mathbb{P})$ which takes values in some state space $\mathcal{Z}$. The parameter t is interpreted as time, the state space $\mathcal{Z}$ will be either a finite or denumerable set or the euclidean space $\mathbb{R}^d$.

We assume that $(t,\omega) \mapsto Z(t,\omega)$ is jointly measurable. For t fixed, $\omega \mapsto Z(t,\omega)$ is a random variable, namely the random state of the process at time t, whereas for ω fixed, $t \mapsto Z(t,\omega)$ is a measurable function on $\mathbb{R}^+$, a *trajectory* or *path* of the stochastic process. Properties of trajectories such as continuity, piecewise continuity or differentiability are crucial in the analysis of stochastic processes.

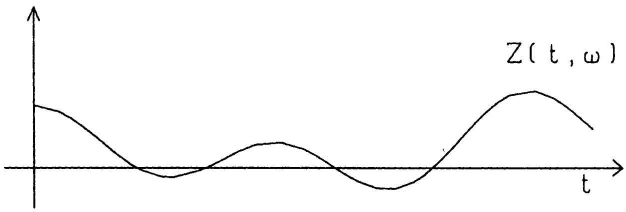

Figure 2.1: A continuous trajectory of some stochastic process

Discrete-Event processes form a special class of stochastic processes. They are characterized by the fact that the trajectories are piecewise constant functions.

2.1 Definition. A *Discrete-Event process* is a stochastic process $(Z(t,\omega))_{t\in\mathbb{R}^+}$ having trajectories which are piecewise constant and continuous from the right

$$Z(t,\omega) = \lim_{s\downarrow t} Z(s,\omega),$$

G. C. Pflug, *Optimization of Stochastic Models*

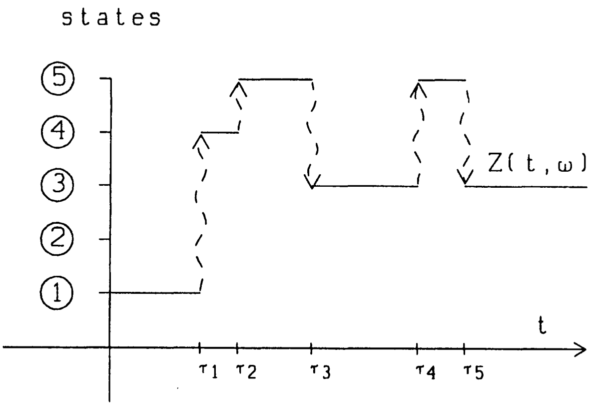

Figure 2.2: A trajectory of a Discrete–Event process

such that each trajectory takes only a finite number of values within each bounded interval of time:

$$\#\{Z(t,\omega) : t \in [a,b]\} \text{ is a.s. finite for } 0 \le a < b < \infty.$$

In the following, we will not write explicitly the random element ω unless it is necessary. Thus we write $Z(t)$ instead of $Z(t,\omega)$, when no confusion may occur.

A Discrete-Event process is indexed by (continuous) time $t \in \mathbb{R}^+$. It may however be described by two sequences of random variables: The sequence of jump times $(\tau_n)_{n=0,1,\dots}$ and the sequence of states (Z_n). The continuous-time process $Z(t)$ is determined by setting

$$Z(t) = Z_n \quad \text{if } t \quad \tau_n \le t < \tau_{n+1}.$$

The differences $\tau_{n+1} - \tau_n$ are called the *sojourn times.* The process $(Z_n)_{n=0,1,\dots}$, which does not carry the information about the sojourn times is called the *embedded process.*

The simplest Discrete–Event processes are those with nonrandom sojourn times equal to 1; $\tau_{n+1} - \tau_n \equiv 1$, i.e. processes which may change their state only at integer times. These processes are called *Discrete-Time processes* and may be written with in the form $(Z(n,\omega))_{n=0,1,\dots}$. They are identical to their embedded process.

The most important class of Discrete-Time processes are the Markov processes with discrete time, which will be discussed in the next section.