Chapter 4

Simulation and sensitivity estimation

4.1 Simulation techniques

We will use the word *simulation* exclusively for the technique to mimic a random process on a computer. Since a computer is a deterministic machine, true randomness cannot be produced. Instead, one uses algorithms, which produce values which are (to a certain extent) indistinguishable from realizations of genuine random processes.

The basic problem of simulation is the generation of i.i.d. Uniform[0,1] variables (see e.g. chapter 3 (in volume 2) of Knuth (1973)). The next problem is to generate i.i.d non-uniform variables with the help of uniform ones (see e.g. the book by Devroye (1986)). The final problem is to generate non i.i.d. random processes (see books by Law and Kelton (1991), Ripley (1987) and Rubinstein (1981,1986)).

We will review briefly the most important techniques for non-uniform random number generation in section 4.1 and the extension of these methods for the generation of derivatives in section 4.2.

4.1.1 Random number generation

Let μ_x be a discrete or continuous probability distribution on $\mathbb{R}$. The parameter x is of no importance at the moment, it is some fixed value. Later, in section

G. C. Pflug, *Optimization of Stochastic Models*

4.2, this parameter will vary and sensitivity with respect to this parameter will be the issue.

We want to generate a random variate Z_x with distribution μ_x. The basic assumption is that a mechanism is available which generates a sequence (U_i) of i.i.d. Uniform[0,1] variables. This is only a dream since none of the known methods like *linear congruential* (Lehmer-Rotenberg), *Tausworthe* or *inverse congruential* produces really such a sequence. But in order to separate the difficulties such an assumption is usually made.

There are hundreds of methods to produce a distribution μ_x out of some basic sequence (U_i) of uniform $[0,1]$ random deviates. Some of them use a fixed number of U_i's, some a variable number. From this rich variety of known generation methods we discuss only four as the most important ones: the transformation method, the acceptance–rejection method, the mixture method and the generation of discrete distributions.

The transformation method

The simplest generation method of a random deviate is found if we may express the variable of interest as a function K_x of one or more uniform random numbers. The *inverse method*, i.e. the generation of a variable with distribution function G_x by $G_x^{-1}(U)$; $U \sim$ Uniform$[0,1]$ is a particular example.

ALGORITHM T

Generate $U_1, U_2, \ldots, U_k$ **from Uniform[0,1]**
$Z_x \leftarrow K_x(U_1, U_2, \ldots, U_k)$
Return Z_x

4.1 Examples. An Exponential(1) variate is generated with $K(u) = -\log(u)$ (inverse method).
A Normal(0,1) variate is generated with $K(u_1, u_2) = \sqrt{-2\log(u_1)}\cos(2\pi u_2)$ (Box–Muller method).

The rejection method

Suppose we want to generate a random variate Z_x from a density $g_x(\cdot)$ and

(1) a random variate S_x from density $k_x(\cdot)$ is available

(2) $\sup_v \frac{g_x(v)}{k_x(v)} < \infty$,