CAN WE BUILD A SUBJECTIVIST STATISTICAL PACKAGE?

Michael Goldstein

Department of Statistics
University of Hull
England

1. INTRODUCTION

This paper concerns the practical implementation of subjectivist theory, and in particular the conceptions of Professor de Finetti, in the form of a statistical package. We begin by briefly considering certain features which distinguish de Finetti's approach from that of standard Bayesian methodologies.

Subjectivist theory can be characterised as the examination of the reasonableness of our modes of thought. In this study "Everything is essentially the fruit of a thorough examination of the subject matter, carried out in an unprejudiced manner, with the aim of rooting out nonsense." What is important is "...the systematic and constant concentration on the unity of the whole, avoiding piecemeal tinkering about, which is inconsistent with the whole; this yields, in itself, something new. "(Both these quotations are from de Finetti (1974:preface)).

This spirit is embodied in the content of de Finetti's work. As an example, he makes expectation, or prevision, fundamental instead of probability, because, once we can free ourselves from historical conventions, there are many advantages, practical, logical and philosophical in this choice. For example, we can now make directly those expectation statements that we require without exhaustive consideration of limiting partitions. De Finetti repeatedly emphasises the need to remain within the bounds of realism. He expresses this as a fundamental requirement as follows. "The fact is the possibility of expressing all that can legitimately be said by arguing solely in terms of the events (and random quantities) whose prevision is known. That is to say, without leaving the linear ambit determined by the latter, without imagining already present a probability distribution over larger ambiets, those in which the extension is possible, albeit in an infinite number of ways. The criterion lies in the commitment to systematically exploiting this fact; the commitment considered as the expression of a fundamental methodological need in the theory of probability (at least in the conception which we here maintain). All this is not usually emphasised." (de Finetti (1974)).

As a second example, statistical models are constructed not in terms of unobservable (and ultimately undefinable) parameters, but instead through the notion of exchangeability, so that any such model can be explicated purely in terms of simple, verifiable statements of uncertainty about...
observable quantities. As de Finetti (1975:p.221) writes: "If we step out of this ambit, we not only find ourselves unable to reach out to something more concrete, but we tumble into an abyss, an illusory and metaphysical kingdom, peopled by Platonic shadows."

As a final example, in the Bayesian approach prior probabilities are transformed into "posterior" probabilities by Bayes theorem. However Bayes theorem actually evaluates "conditional" probabilities. Conditional beliefs are those based on "assumed knowledge", and are expressed as bets to be made now but to become operative only if certain events actually occur. Posterior beliefs are based on acquired knowledge, and are expressed as bets made after certain events are seen to occur, at terms which then seem fair. Logically, these are different concepts. De Finetti (1972:p.194) summarises the interpretation of conditioning as follows. "What emerges is this: only the predictive interpretation (according to which H is a proposition assumed, not acquired) is free of inextricable perplexities."

Each of the above quotations relates essentially to the difference in spirit between a full subjectivist formulation and the Bayesian implementation. In most Bayesian analyses it does not seem to matter whether beliefs are elicited in terms of previsions or probabilities, whether we view probability models as constructed from exchangeability arguments or from genuine beliefs in unknown parameters, whether we are dealing with conditional probabilities or posterior probabilities and so forth. This is because the language and ideas of "belief revision" are being used, in the main, to describe and support the process of "data analysis". However Bayesian methods appear to be tackling a quite different problem, namely how should you "reasonably" modify your beliefs in the light of (statistical) data.

To develop a subjectivist approach to the problems of learning from evidence, we must return to the roots of the theory and decide which elements are essential, which are peripheral and which are, possibly, wrong. This paper describes one such subjectivist approach, taking as a starting point the foundations set out in de Finetti (1974,1975). We shall concentrate upon general issues, as basic disagreements about the content and purpose of the theory can only be resolved when we view the whole structure in a unified manner. Thus we must clarify the substantive content of the theory before we can describe the technical content of our methods.

The plan of the paper is as follows. In section (2), we suggest informal criteria for a subjectivist statistical package and set out various reasons why fully subjectivist approaches are needed. In section (3), we describe informally our approach to such a package, with particular emphasis on the role of exchangeability, the nature of inference and the organising principles for input and output. In section (4), we describe our first steps towards implementing these ideas. Finally, in section (5), we make very brief concluding comments relating to the problems and potential for the general development of subjectivist packages.

2. WHAT IS A SUBJECTIVIST STATISTICAL PACKAGE AND WHY SHOULD WE BUILD ONE?

2.1 What Is A Subjectivist Statistical Package?

A subjectivist package is primarily concerned with the judgements, however expressed, of an individual. The package is intended to facilitate the reasonable elicitation and modification (at least in part by "statistical" data) of beliefs by subjectivist principles. Let us identify some features which would distinguish a full subjectivist package from a package which applies subjectivist ideas in an informal fashion, for example as data-analytic tools. (Thus, we will not emphasise the features that both types