Supersymmetric quantum field theories are those Lagrangian field theories which possess a set of conserved spinor currents:

\[ J^i_{\mu\alpha}(x) , \, \gamma^\mu J^i_{\mu\alpha}(x) = 0 \quad (\alpha = \text{spinor index}) \]

\[ (\mu = \text{vector index}) \]  

\[ i = 1 \ldots N \]  

(1)  

such that the corresponding charges

\[ Q^i_{\alpha} = \int d^3x \, J^i_{0\alpha}(x) \quad (Q^i_{\alpha} \text{ Hermitian}) \]  

(2)  

are constant of motion

\[ \frac{d}{dt} Q^i_{\alpha} = 0 \]  

(3)  

The basic anticommutation relation

\[ \{Q^i_{\alpha}, Q^j_{\beta}\} = (\gamma^\mu C)_{\alpha\beta} P^i_{\mu} \delta^{ij} + (\text{central charges}) \]  

(4)  

(C charge conjugation matrix)

shows the deep interplay between the supersymmetry and the space-time Poincare symmetry. The operator \( P^i_{\mu} \) is the displacement operator of the Poincare algebra. The interactions and particle content of supersymmetric theories strongly depend on the total number of spinor charges \( N \).
From the representations of N-extended supersymmetry on massless one-particle states one gets the following multiplet structure

\[ \lambda_{\text{max}}, \lambda_{\text{max}} - \frac{1}{2}, \ldots, \lambda_{\text{max}} - \frac{K}{2}, \ldots, \lambda_{\text{max}} - \frac{N}{2} \]  

(5)

\[ + \text{(PCT conjugate states with } \lambda_{\text{max}} + \frac{N}{2} - \lambda_{\text{max}}) \]

in which \( \lambda_{\text{max}} \) is the massless state of maximal (positive) helicity inside the multiplet. From Eq. (5) it follows

\[ \lambda_{\text{max}} \geq \frac{N}{4} \left( \frac{N+1}{4} \right) \text{ for odd } N \]  

(6)

so that renormalizable theories (\( \lambda_{\text{max}} \leq 1 \)) require \( N \leq 4 \). If one includes gravitational interaction with the graviton being identified with the state \( \lambda_{\text{max}} = 2 \), then one gets the bound \( N \leq 8 \).

Supersymmetric theories improve the ultraviolet behavior of conventional quantum field theories. There are examples of ultraviolet finite four-dimensional quantum field theories (N=4 Yang-Mills and N=2 Yang-Mills with soft-breaking terms\(^2\)). There are also examples of supersymmetric theories of gravity (supergravity) which have a milder quantum behavior than Einstein theory, and N=8 supergravity is considered to be the most appealing candidate theory for a completely unified theory of all fundamental interactions\(^3\).

Supersymmetry seems to be the only known symmetry which protects scalar masses from huge radiative corrections due to the so-called non-renormalization theorem\(^4\) (naturalness principle of small numbers). This fact led to the speculation that supersymmetry may solve the so-called hierarchy problem of conventional G.U.T.'s, i.e., the stability of \( M_H/M_X \ll 1 \) in perturbation theory\(^5-6\). An outstanding question is how to embed the observed fundamental interactions in supersymmetric theories.