ATTENUATION CORRECTION USING MELLIN TRANSFORM

Jean-Marie Nicolas and Jean-Luc Bernatets
Laboratoires d'Electronique et de Physique appliquée
3, avenue Descartes, 94450 Limeil-Brevannes, France

INTRODUCTION

It has been shown\(^1\) that the incidence of attenuation in ultrasonic propagation within tissue can be considered as an integral transformation of acoustical signals. This transformation is known among the mathematician as a "Mellin-convolution". With respect to Mellin-convolution, Mellin transformation plays the same role as Fourier transformation does with respect to usual convolution. That is, it turns Mellin-convolution of two functions into a straightforward product of their Mellin transforms.

The Mellin transform is used to compensate for the effects of the linear attenuation on echographic signals by Mellin spectra division.

The results of this procedure are validated by measuring, with the help of methods involving both spectral-centroid down shift and narrow band signal decay\(^2\), the apparent attenuation on processed A-lines originating from phantoms.

Images are shown that evidence the effects of attenuation compensation in two respects:

- in amplitude: compensation of general brightness decay;
- in resolution: by restoring the high frequency components of the echographic signal in the far field, the resolution in the distal part of the scans is improved.

Such images show that the method which is proposed in this paper is much more than a conventional TGC.
I. MODELS AND METHODS

I.1. Mellin-convolution and Mellin transformation: application for attenuation compensation

The Mellin-convolution of two functions $f_1$ and $f_2$ is a function $f_3$ defined as:

$$f_3 = f_1 \ast M f_2 = \int_0^\infty f_1(u) e^x \frac{du}{x} = \int_0^\infty f_1(x) f_2(u) \frac{du}{u}$$  \hspace{1cm} (1)

The name of "Mellin-convolution" stems from its relation to the Mellin transformation which is defined as:

$$M(f(x))(s) = \int_0^\infty f(x) x^{s-1} dx = F(s)$$  \hspace{1cm} (2)

or conversely:

$$M^{-1}(F(s))(x) = \frac{1}{2i\pi} \int_{a-i\infty}^{a+i\infty} F(s) x^{-s} ds = f(x)$$  \hspace{1cm} (2')

In terms of Mellin transformed, (1) can be rewritten as:

$$M(f_3) = M(f_1) \cdot M(f_2)$$  \hspace{1cm} (3)

The attenuation of ultrasound in biological tissue has a linear frequency dependence, i.e. its transfer function can be written as:

$$A(l,f) = e^{-2\pi l |f|} \frac{1}{c}$$  \hspace{1cm} (4)

at depth $l$, $c$ being the velocity of the medium.

It has already been shown\(^1\) that Mellin-convolution is an effective tool to model the effects of such an attenuation (see also Appendix I). Let us recall the main results. The velocity potential $\psi$ can be deduced from the velocity potential which would be obtained in a non-attenuating medium: $\psi_{NA}$ with the help of a Mellin-convolution by a function depending only on the attenuation:

$$\psi(t) = \psi_{NA}(t) \ast h(t) \leftrightarrow M(\psi) = M(\psi_{NA}) . M(h)$$

with:

$$h(t) = \frac{1}{\pi} \frac{c}{c^2 + (t-1)^2}$$  \hspace{1cm} (5)

I.2. Ultrasonic echographic signal modelling with the help of the Mellin transform

In the case of a single target located at $r$, the backscattering function of which is $u(r,t)$, the echographic signal can be modelled as\(^3\):

$$s(t) = e(t) \ast i_t^c(t) \ast \psi_t^c(r,t) \ast u(r,t) \ast \psi_t^c(r,t) \ast i_t^r(t)$$  \hspace{1cm} (6)

with:

- $e(t)$: the voltage that drives the transducer;
- $i_t^c(t)$: the acousto-electric response of the transducer, in transmit mode;
- $i_t^r(t)$: the acousto-electric response of the transducer, in receive mode.

Putting $w(t) = e(t) \ast i_t^c(t) \ast i_t^r(t)$, we obtain: