MEASUREMENT AND SIMULATION OF THE SCATTERING OF ULTRASOUND BY
PENETRABLE CYLINDERS

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INTRODUCTION

Scattering of ultrasound occurs when a propagating wave encounters variations in some intrinsic property, such as density compressibility or absorption, of the propagation medium. Scattering is manifested as a change in the spatial distribution (and, in the case of absorption, the total amount) of the ultrasonic energy. There are four distinct components to be considered in any scattering problem. These are:
(i) The propagation process. A description of the physics of the propagation process is embodied in the wave equation which can be regarded as the propagation model.
(ii) The spatial distribution of acoustic properties of the medium, i.e., the 'object', which in this treatment we call $F$.
(iii) The insonifying wave, $\psi_0$. In the absence of the object $\psi_0$ is the only field existing. However, in the presence of the object scattering causes an additional field called
(iv) the scattered field, $\psi_s$. The actual field present at any point is termed $\psi$ and is the sum

$$\psi = \psi_0 + \psi_s$$

In the so-called direct scattering problem, the scattered field is related to the other components by:
where $G$ is an operator derived from the propagation model. In comparison, ultrasound diffraction tomography is an application of inverse scattering in which information about the object is gained from knowledge of the incident and scattered fields using an inverse operator, $G^{-1}$, also derived from the propagation model. So for inverse scattering we have

$$F = G^{-1}\{\psi_o, \psi_s\} \quad (3)$$

This paper presents some results of a study on the direct scattering problem (Eq. 2). However, due to the intimate relationship between direct and inverse scattering as apparent from Eqs. (2) and (3), the results also have relevance for inverse scattering. In fact, the motivation for the study came from a need to test some of the assumptions we make in diffraction tomography experiments, i.e., most importantly, to verify that we can indeed faithfully measure in the laboratory the data predicted by our theoretical model. To achieve this aim, the scattering of plane ultrasound waves by penetrable circular cylinders was investigated both theoretically and experimentally. This particular geometry was chosen since it is possible to compute an exact solution for the direct scattering problem.$^{1,2}$ Similar comparisons already appear in the literature.$^{3,4}$ However, the agreement between theory$^4$ and experiment$^3$ was poor and furthermore, the early work investigated the magnitude of the scattering in the far-field only.

For our results to have relevance for diffraction tomography, we investigated the complex amplitude (i.e., both magnitude and phase) of forward scattering in the near field. Although scattering from a single cylindrical scatterer immersed in an otherwise homogeneous medium is a somewhat specialized case, it is perhaps not too far removed from, say, a tumor embedded in soft tissues for the results to have practical significance. Accordingly, we studied cylinders whose properties were close to what might be encountered in clinical imaging. The acoustic properties did not differ greatly from water (i.e., the scatterers were tenuous) and the diameters were a few dozen wavelengths across. To further extend previous work$^3,4$ we studied the two-dimensional distribution of scattering in a plane normal to the cylinder axis, the field within the cylinder, and the effects of absorption. Note that we assume the sign convention $\exp(-j\omega t)$ for temporal waveforms meaning that the phase shifts we report are inverted with respect to 'reality' in all results.