DESCRIBING THE LONG TERM NONLINEARITY OF PLASTICS

O. S. Brüller

Institute -A- of Mechanics
Technical University of Munich
Arcisstr. 21, D - 8000 Munich 2

INTRODUCTION

A good method for the mathematical description of the response of a polymer under long term loading conditions (such as creep or stress relaxation) is provided by the use of finite exponential series. In the nonlinear viscoelastic range, the coefficients of the series are dependent upon the level of the applied load. The knowledge of this dependence enables the description of the nonlinear behavior of the material even for more complicated loading histories.

STRESS RELAXATION IN THE LINEAR VISCOELASTIC RANGE

The stress response $\sigma(t)$ of a viscoelastic material to a constant uniaxial strain applied under unchanged environmental conditions at time $t=0$ is a function of time of the form:

$$\sigma(t) = \epsilon_{\text{app}} E(t)$$

where $E(t)$ is the uniaxial relaxation modulus of the considered material and $\epsilon_{\text{app}}$ the applied strain.

In the linear viscoelastic range the relaxation modulus is a function of time only and can be approximated by the relation:

$$E(t) = \left( E_0 + \sum_{i=1}^{n} \frac{\epsilon_i}{t^i} \right)$$

G. Astarita et al. (eds.), *Polymer Processing and Properties*
© Plenum Press, New York 1984
In order to obtain an accurate approximation, it is sufficient to choose the \( m \) discrete "relaxation times" \( \tau_i \) one per decade in the time interval of the experiment\(^1,\)\(^2\). The \( m+1 \) material parameters \( E_i \) can be computed by simple collocation\(^1\) or by using least squares as shown here (see also\(^3\)).

The sum of squares of the differences between the mathematical approximation (2) and the \( n \) experimentally determined values of the relaxation modulus \( E^*(t_j) \) is:

\[
H = \sum_{j=1}^{n} (E^*(t_j) - (E_0 + \sum_{i=1}^{m} E_i e^{-\frac{t_j}{\tau_i}}))^2
\]

(3)

Its minimum is obtained if:

\[
\frac{\delta H}{\delta E_k} = 0 \quad (k = 0, 1, 2, \ldots m)
\]

(4)

The following linear system of \( m+1 \) equations results:

\[
[A^T][A][E] = [A^T][B]
\]

(5)

with:

\[
[A] = \begin{bmatrix}
1 & e^{-t_1/\tau_1} & e^{-t_2/\tau_1} & \cdots & e^{-t_m/\tau_1} \\
1 & e^{-t_1/\tau_2} & e^{-t_2/\tau_2} & \cdots & e^{-t_m/\tau_2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & e^{-t_1/\tau_m} & e^{-t_2/\tau_m} & \cdots & e^{-t_m/\tau_m}
\end{bmatrix}
\]

(6)

\[
[E] = \begin{bmatrix}
E_0 \\
E_1 \\
\vdots \\
E_m
\end{bmatrix}
\]

(7)

\[
[B] = \begin{bmatrix}
E^*(t_1) \\
E^*(t_2) \\
\vdots \\
E^*(t_n)
\end{bmatrix}
\]

(8)

and \( [A] \) as transposed matrix of \( [A^T] \).