EXTENSION OF NEWMAN'S NUMERICAL TECHNIQUE TO
PENTADIAGONAL SYSTEMS OF EQUATIONS

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ABSTRACT

A finite difference technique accurate to $O(h^4)$ for a set of coupled, nonlinear second-order ordinary differential equations is presented. It consists of extending Newman's technique for coupled, tridiagonal equations to a set of coupled pentadiagonal equations. The method can be used to reduce the number of node points needed for a given accuracy or to maintain accuracy to $O(h^2)$ for boundary value problems that include multiple interior regions with continuity of flux of field variables from one region to the next (i.e., interior boundary points with derivative boundary conditions).

Introduction

Newman (1968) (see also White (1978) presented a numerical solution technique for solving two-point boundary value problems which
involve coupled, nonlinear, ordinary differential equations. Newman's method consists of collapsing a large sparse matrix into a tridiagonal matrix with matrix elements (see Newman (1973) for the code, subroutine BAND(J)). In essence, this tridiagonal matrix results from the three-point finite difference approximations used to represent the derivatives to $O(h^2)$. Newman (in White et al. (1975)) extended his method to handle five-point difference approximations to obtain accuracy to $O(h^4)$ for two-point boundary value problems or to handle continuity of flux conditions at interior boundary points to maintain accuracy to $O(h^2)$ for problems with multiple regions of interest. The purpose of this paper is to present the matrix equations that arise when applying the technique to a system of nonlinear ordinary differential equations. It is worth noting that solution of these equations does not require storing any of the jacobian elements but instead only one three dimensional array. In addition, an alternative formulation of the nonlinear problem is presented.

Method

A set of nonlinear, coupled, second order ordinary differential equations can be written as:

$$f_i = \sum_{k=1}^{n} u_{i,k}(x,c) \frac{d^2c_k}{dx^2} + v_{i,k}(x,c) \frac{dc_k}{dx} + w_{i,k}(x,c) c_k(x) = 0 \quad (1)$$

where $c$ represents $[c_1 \ c_2 \ldots \ c_n]^T$ and $i$ represents the equation number and varies from 1 to $n$ (the number of unknowns or field variables) for a complete set of equations. (Equation 1 could also be nonlinear in the derivatives, but here the simpler form was chosen for illustration). The governing nonlinear boundary conditions for this set of differential equations can also be written in general form: