THERMAL STRESS FRACTURE IN ELASTIC-BRITTLE MATERIALS

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ABSTRACT

The uncoupled thermoelastic brittle fracture problem is discussed in terms of the types of stress fields produced by surface heating or cooling and the generic characteristics of the thermally generated stress intensity factors. Examples of experimental measurements and numerical calculations are given to demonstrate these general characteristics.

INTRODUCTION

Thermal stresses are usually generated by one of two mechanisms: the creation of temperature variations within a body because of imposed surface conditions; temperature distributions caused by the generation of heat by internal thermal sources, such as joulean dissipation, or by the conversion of mechanical energy to thermal energy such as that found in stress waves or by fatiguing processes. The solution of a problem which involves the conversion of mechanical to thermal energy requires the simultaneous solution of the structural and thermal equations but except for thermally generated stress waves or strong dissipative effects, as found in plastic forming, it is rare to have to consider the coupled problem. For brittle materials and for temperatures caused by surface imposed conditions such coupled effects are unlikely.

In this paper we discuss the general characteristics of thermal stresses and the mode I and II stress intensities which are produced by such stresses in two dimensional structures. No attempt is made to survey the entire field or even to establish the details of the calculations or of the solutions. Rather the desire is to impart
to the reader a feeling for the spatial and temporal behavior of the thermal stress fields, particularly in those areas where thermal and mechanical stresses differ most. For detailed solutions the reader is directed to references 1-4.

For a region V with surfaces S, the pertinent equations which model the uncoupled thermal stresses are the first law of thermodynamics and conservation of momentum.

a) For the temperature, the use of Fourier's relationship \( q_i = -k \frac{\partial T}{\partial x_i} \) with the first law gives the field equation

\[
\frac{\partial pcT}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + Q \]  

(1a)

with an initial condition

\[
T(x,y,z,0) = f(x,y,z) \]  

(1b)

and boundary conditions of

\[
T = g(x,y,z) \text{ on } S_T \text{ for prescribed temperature} \]  

(1c)

\[
k_n \frac{\partial T}{\partial n} = q(x,y,z) \text{ on } S_q \text{ for prescribed heat flux} \]  

(1d)

\[
k_n \frac{\partial T}{\partial n} = h(T - T) \text{ on } S_c \text{ for convective cooling} \]  

(1e)

b) The elastic displacements and stresses for static conditions are given by the equilibrium equations

\[
\frac{\partial \sigma_{ij}}{\partial x_i} + X_j = 0 \]  

(2a)

with boundary conditions of the form

\[
\begin{align*}
\sigma_{nn} &= t_t(x,y,z) \quad \left\{ \text{on } S_t \right\} \\
\sigma_{ns} &= t_s(x,y,z) \quad \left\{ \text{on } S_s \right\}
\end{align*} \]  

(2c)

with the constitutive equations for linear elastic materials of the form