Since the time when the study of relations between the various critical indices was systematized, these indices have been classified into groups. First, I remind you of some notation. If $\chi$ is the magnetic susceptibility, $M$ the magnetization, $C_H$ the specific heat at constant magnetic field, and $\xi$ the correlation length, then near the critical point, temperature, $T = T_c$, and magnetic field, $H = 0$, for an Ising model on a $d$-dimensional, rigid, regular space-lattice we expect, $T > T_c$, $H = 0$,

\[ \frac{\partial^2 \chi}{\partial H^2} = B_+(T-T_c)^{-\gamma-2\Delta}, \quad C_H \propto (T-T_c)^{-\alpha}, \]

\[ T = T_c, \]

\[ M \propto H^{1/\delta}, \quad \langle \sigma_0 \sigma_r \rangle \big|_{H=0} \propto r^{-d+2-\eta} \]

where $\langle \sigma_0 \sigma_r \rangle \big|_{H=0}$ is the spin-spin correlation function between a spin $\sigma$ at the origin and one at $r$ in zero magnetic field.

\[ T < T_c, \quad H = 0, \]

*Work supported in part by the U.S. D.O.E.
\[ \chi \approx A_{-}(T_c - T)^{-\gamma'}, \quad \xi \approx D_{-}(T_c - T)^{-\nu'} \]

\[ -\frac{\partial^2 \chi}{\partial \mu^2} \approx B_{-}(T_c - T)^{-\gamma' - 2\Delta'}, \quad C_H \propto (T_c - T)^{-\alpha'} \]

\[ M \propto (T_c - T)^{\beta}. \quad (3) \]

In terms of this notation, a selection of the relations between the critical indices (\(\alpha, \gamma, \delta, \) etc.) would be:

single temperature region,
\[ \alpha' + 2 + \gamma' = 2; \quad (4) \]
critical isotherm plus a single temperature region,
\[ \alpha' + \beta(1+\delta) = 2, \]
\[ \delta = \Delta/(\Delta - \gamma); \quad (5) \]
two temperature regions
\[ \gamma = \gamma', \quad \alpha = \alpha', \]
\[ \Delta = \Delta'; \quad (6) \]
relations involving correlation exponents,
\[ \gamma = (2-\eta)\nu, \]
\[ \gamma' = (2-\eta)\nu'; \quad (7) \]
and relations involving the spatial dimension or hyperscaling,
\[ d\nu = 2 - \alpha, \]
\[ 2 - \eta = d(\delta - 1)/(\delta + 1), \]
\[ 2\Delta = d\nu + \gamma. \quad (8) \]

On the numerical evidence, the hyperscaling relations (8) were the least well supported and those of (6) suffered initially from the weakness of the accuracy in the \(T < T_c\) numerical results. Many of these relations have been proven to be rigorous inequalities, e.g., \(^2\)