INTRODUCTION

We present a complementary approach to the time dependent scattering theory described by V. Enss for one-body Schrödinger operators. Roughly speaking the stationary theory is concerned with those objects you read about in textbooks on quantum theory like scattering waves and amplitudes. The starting point of the old theory is not here an asymptotic condition for large times, as for wave-operators of V. Enss lecture, but rather for large distances. It is known as "Sommerfeld radiation condition" and leads, when incorporated in the Schrödinger equation as a "Cauchy condition at infinity", to the celebrated Lippman–Schwinger equation. In the more recent abstract stationary theory some generalized form of the Lippman–Schwinger equation plays the basic role; solving this equation leads to a linear map between generalized eigenfunctions of the perturbed and unperturbed operators. This map is the "section" at fixed energy of the wave-operator from the time dependent theory. Although the radiation condition does not appear explicitly in this formulation it can be shown to hold a posteriori in a variety of situations, thus restoring the link with physical theories. A general approach to the radiation condition for a large class of Partial Differential operators is described in the work of S. Agmon and L. Hörmander; we will mention some of their results here.
In these lectures I will describe an abstract framework for the stationary theory. It is strongly inspired from the Kato-Kuroda theory and also related to the more recent two Hilbert space theories of T. Kato, M. Schecter, or I. Segal. As we will see it allows to incorporate many of the technical progress of these last years. Among them are S. Agmon's "elliptic" a priori estimate method and the geometric methods; these last have appeared as particularly useful for many investigations on the bound state problem for N-body Hamiltonians. They also seem to be promising for the scattering problem.

THE STANDARD FORMULATION OF STATIONARY SCATTERING THEORY FOR THE ONE-BODY QUANTUM PROBLEMS

The basic ansatz of orthodox textbooks on quantum scattering in \( \mathbb{R}^n \) by a local potential \( V(X) \) vanishing at infinity is the existence of a family of solutions \( u^{(k)}(x) \) of the reduced Schrödinger equation:

\[
(-\Delta + V) u^{(k)}_+(x) = 0
\]

which can be decomposed as

\[
(1) \quad u^{(k)}_+(x) = u^{(k)}_0(x) + \mathcal{V}^{(k)}(x)
\]

where \( \mathcal{V}^{(k)}(x) \) is a "scattered wave" and \( u^{(k)}_0(x) = \exp(i k x) \).

By this one means essentially that \( \mathcal{V}^{(k)}(x) \) has the asymptotic form

\[
(2) \quad \mathcal{V}^{(k)}(x) \sim |x|^{1/2} \exp(-i |k| |x|) \Phi(k, \omega)
\]

where \( \omega \) is the angular variable of the particle. This is known as "Sommerfeld Radiation condition" which can be more rigorously stated as

\[
(3) \quad \lim_{R \to \infty} \int_{|x|=R} |\mathcal{V}^{(k)}_+ - i |k| \mathcal{V}^{(k)}_+|^2 \, ds = 0
\]

The function \( f \) is called the "Scattering amplitude" and from it one gets the scattering cross-section

\[
\sigma(k) = \int_S |f(k, \omega)|^2 \, d\omega
\]

where \( S \) is the unit sphere in \( \mathbb{R}^n \).

It has been known for a long time that solutions in class