CHAPTER 5

SPECULATIVE BUBBLES AND CRASHES II: RATIONAL AND SEMI-RATIONAL

"Then we,
As we beheld her striding there alone,
Knew that there never was a world for her
Except the one she sang and, singing, made."
Wallace Stevens, 1947
"The Idea of Order at Key West"

5.1. Self-Fulfilling Prophecies as Rational Bubbles

Mill (1848) and others recognized the existence of self-fulfilling prophecies in speculative markets, but viewed them as fundamentally irrational because they were supposedly deviations from the longer run equilibrium. Keynes (1936, Chap. 12) and Samuelson (1957) were the first to propose that a speculative bubble might possibly be a rational self-fulfilling prophecy in some sense.1

John Muth (1961) explicitly formulated the rational expectations concept to be applied to speculation in agricultural markets and to argue for the unlikeliness of unstable cobweb solutions in such markets. In Muth's formulation, even in a stochastic situation, economic agents will have expectations ("subjective probability distributions of outcomes") for a given information set that will be distributed with a mean equal to the predictions of the correct theory ("objective probability distributions of outcomes").

The first formal models with rational bubble-type solutions arose in discussions of instability and indeterminacy in perfect foresight growth models, the so-called "Hahn problem" of "saddlepoint instability." Frank Hahn (1966) showed that in growth models with heterogeneous capital goods and a constant savings rate there could exist an infinite set of possible equilibrium outcomes. The paths diverging from the maximum balanced-growth steady-state equilibrium (turnpike) can be viewed as bubble-like. Samuelson (1967a) further analyzed this model and noted explicitly the link with the possibility of an infinity of "explosive" self-fulfilling prophecies.

It was realized that these arguments extended to cases where capital was homogeneous, but in which there was an alternative asset which could be accumulat-
Sidakauski (1967) presented a model of perfect foresight with respect to the rate of inflation in which the rate of inflation negatively affects the demand for money. This model allows for an infinite set of possible price paths consistent with a single, constant, rate of growth of the money supply, basically a different path for every possible initial price level. This model and its connection to the Hahn-Samuelson saddlepoint problem was examined in more detail by Burmeister and Dobell (1970).

Thus from the very beginning the "New Classical" school was shadowed by the spectre of multiple rational expectations equilibria with most of them being bubbles. The problem only became worse when it became clear that such solutions could still appear in utility-maximizing, perfect foresight, infinite-horizon models with only one asset. Brock (1974) showed this for the case where the asset is money, and Calvo (1978) showed it for a non-monetary economy where the asset is land.

Several efforts to exorcise the spectre of multiple rational expectations bubble equilibria have been attempted. Sargent and Wallace (1973) assumed that the response to any unanticipated monetary shock would be a discontinuous change in the price level putting the system back on the presumably unique, non-bubble equilibrium path. That such a jump could be relied upon to resolve the problem was called into doubt when Black (1974) showed that in such a model there were an infinite set of possible such jumps with following price paths that were consistent with perfect foresight.

Bennett McCallum (1983) proposed an assumption that agents' forecast rules involve the minimal set of state variables with solution formula valid for all admissible parameter values. Such an assumption will achieve the desired result from McCallum's perspective for linear models, since bubbles generally appear due to "extra" variables entering forecast rules, namely expected non-zero rates of change of some prices or other variables. In short it assumes away the possibility of self-fulfilling prophecies, but other than a general appeal to Occam's Razor there is no a priori reason why such an assumption should be generally true, a point McCallum grants. He further argues that the problem of non-uniqueness of solutions is not due to any inherent difficulty with the rational expectations assumption, but, "is simply an inescapable aspect of dynamic models involving expectations."

A more recent effort to deal with the non-uniqueness problem in rational expectations models is due to George Evans (1985). He considers the stability of expectations functions, if an expectations function is disturbed, will the system return to it or not? He argues that for a fairly simple adjustment rule, or learning process, only the non-bubble solution will be stable. This would imply some greater tendency for convergence to that particular solution over all than for possible bubble solutions.

Evans recognizes that this is only fully true for certain parameter values of his "revision rule." In other cases bubble solutions may have a degree of expectational stability. In particular Woodford (1990) has shown broad cases for which learning processes could converge on sunspot and hence on bubble equilibria.