Since the experimental techniques for creating phonon pulses and also nearly monochromatic phonon waves are developed so far, the scattering of phonons due to crystal defects gained much interest. In the theoretical formulation of these problems many difficulties arise when the defect is of complicated nature, e.g. when the coupled electronic states are degenerate. Such a situation occurs when we study the phonon scattering by a Jahn-Teller defect.

In the present investigation we consider a trigonal Jahn-Teller centre of E-e nature. The Hamiltonian is given by \( \mathbf{H} = \sum \frac{1}{2} (a_i^* a_i + a_i a_i^*) \sum \omega_k (b_{ik}^* b_{ik} + b_{ik} b_{ik}^*) \) for \( \mathbf{n} = 1 \). 

The \( b_{ik}^*, b_{ik} \) (i = 1,2, index of degeneracy) are the creation and annihilation operators of the degenerate phonon modes, whereas the \( a_i^*, a_i \) are the corresponding electronic operators. The dynamic of the local electronic system is governed by a quasi-spin algebra. The spin operators are defined by:

\[
\mathbf{\sigma}_z = \mathbf{\sigma}_n = \frac{1}{2} \left( a_1^* a_1 - a_2^* a_2 \right) \\
\mathbf{\sigma}_x = \mathbf{\sigma}_2 = \frac{1}{2} \left( a_1^* a_2 + a_2^* a_1 \right) \\
\mathbf{\sigma}_y = \mathbf{\sigma}_3 = \frac{1}{2i} \left( a_1^* a_2 - a_2^* a_1 \right).
\]
To derive an expression for the inverse life-time of phonons of the sort \( (i,k) \), we use a Green's function formalism first developed by Klein for a two-level system.\(^1\) The electronic part of Klein's system is described by the same quasi-spin algebra as in our model. In this situation the phonon-phonon Green's function can be traced back to Green's functions connecting the introduced quasi-spin operators. The relaxation time get the form

\[
\tau_{i,k}^{-1}(\omega+i\epsilon) = \omega_i^{-2} \lim_{\epsilon \to 0^+} \text{Im} \, g_i(\omega+i\epsilon),
\]

where the Green's function \( g_i(\omega) \) reads

\[
g_i(\omega) = \left< \left< \hat{\sigma}_i; \hat{\sigma}_i \right> \right>(\omega) \quad (i=1,2).
\]

For the calculation of the "spin-spin" Green's functions we employ an exponential transformation of the type

\[
\tilde{H} = e^{-S} He^S = H + [H,S] + \frac{1}{2!} [[H,S],S] + \ldots,
\]

which diagonalizes the Hamiltonian mainly. In the E-e model \( S \) is found to be\(^2,3\)

\[
S = \sum_{k} \frac{K(k)}{\omega_k^2} \left\{ G_k^2 (b_{ik} - b_{ik}^+) + G_k^x (b_{2k} - b_{2k}^+) \right\}.
\]

It could be shown that this transformation yields to exact results in the very extremal coupling regions. But also in the intermediate coupling cases the transformation leads to good results as proven by variational calculations of the eigenvalues using a great number of basis functions.

The "spin-spin" Green's function can be more easily calculated in the new coordinations. Therefore we must transform all operators, e.g.

\[
\hat{\sigma}_i \rightarrow \hat{\sigma}_i = e^{-S} \hat{\sigma}_i e^S.
\]

By a fourth order RPA-decoupling the interesting Green's functions are given by\(^4\)

\[
\left< \left< \hat{\sigma}_i; \hat{\sigma}_i \right> \right>(\omega) = \{-2\omega^2 \sum_{k} \frac{K(k)}{\omega_k^2 - \omega_k^2} \left< \hat{\sigma}_i \left( \hat{\sigma}_i + \hat{\sigma}_i^+ \right) \right> \}

\times \left\{ \omega^2 - 4 \sum_{k} \frac{K(k)^2}{\omega_k^2 - \omega_k^2} \left< \hat{\sigma}_i \right> \sum_{k} K(k) \left< \hat{\sigma}_i + \hat{\sigma}_i^+ \right> + \right.

\left. 4 \omega \sum_{k} \frac{K(k)^2}{\omega_k^2 - \omega_k^2} - 4 \omega^2 \sum_{k} \frac{K(k)}{\omega_k^2 - \omega_k^2} \left< \left( \hat{\sigma}_i + \hat{\sigma}_i^+ \right) \sum_{k} K(k) \left( \hat{\sigma}_i + \hat{\sigma}_i^+ \right) \right> \right\}^{-1}
\]

\(< \ldots >_0 \) denotes the thermal average.