Applied Mathematics Is Bad Mathematics

Paul R. Halmos

It isn't really (applied mathematics, that is, isn't really bad mathematics), but it's different.

Does that sound as if I had set out to capture your attention, and, having succeeded, decided forthwith to back down and become conciliatory? Nothing of the sort! The "conciliatory" sentence is controversial, believe it or not; lots of people argue, vehemently, that it (meaning applied mathematics) is not different at all, it's all the same as pure mathematics; and anybody who says otherwise is probably a reactionary establishmentarian and certainly wrong.

If you're not a professional mathematician, you may be astonished to learn that (according to some people) there are different kinds of mathematics, and that there is anything in the subject for anyone to get excited about. There are; and there is; and what follows is a fragment of what might be called the pertinent sociology of mathematics: what's the difference between pure and applied, how do mathematicians feel about the rift, and what's likely to happen to it in the centuries to come?

What is it?

There is never any doubt about what mathematics encompasses and what it does not, but it is not easy to find words that describe precisely what it is. In many discussions, moreover, mathematics is not described as a whole

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but is divided into two parts, and not just in one way; there are two kinds of mathematics according to each of several different systems of classification.

Some of the dichotomies are well known, and others less so. Mathematics studies sizes and shapes, or, in other words, numbers (arithmetic) and figures (geometry); it can be discrete or continuous; it is sometimes finite and sometimes infinite; and, most acrimoniously, some of it is pure (useless?) and some applied (practical?). Different as these classification schemes might be, they are not unrelated. They are, however, not of equal strengths; the size-shape division, for instance, is much less clear-cut, and much less divisive, than the pure-applied one.

Nobody is forced to decide between vanilla ice cream and chocolate once and for all, and it is even possible to mix the two, but most people usually ask for the same one. A similar (congenital?) division of taste exists for mathematicians. Nobody has to decide once and for all to like only algebra (discrete) or only topology (continuous), and there are even flourishing subjects called algebraic topology and topological algebra, but most mathematicians do in fact lean strongly toward either the discrete or the continuous.

Squares and spheres

It would be a shame to go on and on about mathematics and its parts without looking at a few good concrete examples, but genuine examples are much too technical to describe in the present context. Here are a couple of artificial ones (with some shortcomings, which I shall explain presently).

Suppose you want to pave the floor of a room whose shape is a perfect square with tiles that are themselves squares so that no two tiles are exactly the same size. Can it be done? In other words, can one cover a square with a finite number of non-overlapping smaller squares all of which have different side-lengths? This is not an easy question to answer.

Here is another puzzle: if you have a perfect sphere, like a basketball, what's the smallest number of points you can mark on it so that every point on the surface is within an inch of one of the marked ones? In other words, what's the most economical way to distribute television relay stations on the surface of the globe?

Is the square example about sizes (numbers) or shapes (figures)? The answer seems to be that it's about both, and so is the sphere example. In this respect the examples give a fair picture; mixed types are more likely to occur (and are always more interesting) than the ones at either extreme. The examples have different flavors, however. The square one is more nearly arithmetic, discrete, finite, pure, and the one about spheres leans toward being geometric, continuous, infinite, applied.