A STUDY OF THE SHORT WAVE COMPONENTS IN COMPUTATIONAL ACOUSTICS

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ABSTRACT

The feasibility of performing direct numerical simulations of acoustic wave propagation problems has recently been demonstrated by a number of investigators. It is easy to show that the computed acoustic wave solutions are good approximations of those of the exact solutions of the linearized Euler equations as long as the wavenumbers are in the long wave range. Computed waves with higher wavenumber, or the short waves, generally have totally different propagation characteristics. There are no counterparts of such waves in the exact solutions. The short waves are contaminants of the numerical solutions. The characteristics of these short waves are analyzed here by group velocity consideration. Numerical results of direct simulations of these waves are reported. To purge the short waves so as to improve the quality of the numerical solution, it is suggested that artificial selective damping terms be added to the finite difference scheme. It is shown how the coefficients of such damping terms may be chosen so that damping is confined primarily to the high wavenumber range. This is important for then only the short waves are damped leaving the long waves basically unaffected. The effectiveness of the artificial selective damping terms is demonstrated by direct numerical simulations involving acoustic wave pulses with discontinuous wave fronts.

1. Introduction

Linear acoustic waves are governed by the linearized Euler equations. These waves are nondispersive, nondissipative and isotropic. They propagate at exactly the speed of sound. To calculate these

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waves computationally, say, by using a finite difference scheme it is not at all clear that the numerical solutions will retain these characteristics. On the contrary it is well known that finite difference approximation invariably introduces numerical dispersion, dissipation and mesh anisotropy (Vichnevetsky and Bowles, 1982 and Trefethen, 1982). These problems are most severe for short waves or waves with high wavenumbers. For very short waves inevitably there will be numerical dispersion. Further, spurious grid-to-grid oscillations, referred to as parasite waves (Trefethen, 1982), are often found. These numerical contaminants are most undesirable. They degrade the quality of the numerical solutions. If the fully nonlinear Euler equations are used they might even cause extraneous distortion of the waveform and perhaps nonlinear instability.

The purpose of this paper is two fold. The first objective is to provide a mathematical definition of the short wave components from the wavenumber point of view. A concrete example will be provided to illustrate their uncommon and somewhat unexpected wave propagation characteristics. Since these wave components are not part of the physical solution, the second objective is to introduce a method to eliminate these waves. The long waves with small wavenumbers constitute the useful band of waves for numerical simulation. This portion of the wavenumber spectrum must be protected while the short waves are being purged. To meet this constraint, a method of selective damping is proposed. It will be shown that it is possible to add artificial damping terms to a finite difference scheme which remove primarily the short waves while having negligible effects on the long waves. The effectiveness of this method will be demonstrated by examining the solutions of the linearized Euler equations with discontinuous initial data.

2. The Wavenumber of a Finite Difference Scheme

In a recent paper Tam and Webb (1992) introduced a way to find the effective wavenumber and frequency of a finite difference scheme. Consider the approximation of the first derivative $\frac{\partial f}{\partial x}$ at the $\ell$th node of a uniform grid of spacing $\Delta x$. Suppose $M$ values of $f$ to the right and $N$ values of $f$ to the left of this point are used to form the finite