\[ \begin{align*}
\log 50 &= 2 - x, & \log 500 &= 3 - x \\
\log 5000 &= 4 - x, & \log 50000 &= 5 - x, & \text{&c.}
\end{align*} \]
\[ \begin{align*}
\log 25 &= 2 - 2x, & \log 125 &= 3 - 3x \\
\log 625 &= 4 - 4x, & \log 3125 &= 5 - 5x, & \text{&c.}
\end{align*} \]
\[ \begin{align*}
\log 250 &= 3 - 2x, & \log 2500 &= 4 - 2x \\
\log 25000 &= 5 - 2x, & \log 250000 &= 6 - 2x, & \text{&c.}
\end{align*} \]
\[ \begin{align*}
\log 1250 &= 4 - 3x, & \log 12500 &= 5 - 3x \\
\log 125000 &= 6 - 3x, & \log 1250000 &= 7 - 3x, & \text{&c.}
\end{align*} \]
\[ \begin{align*}
\log 6250 &= 5 - 4x, & \log 62500 &= 6 - 4x \\
\log 625000 &= 7 - 4x, & \log 6250000 &= 8 - 4x, & \text{&c.}
\end{align*} \]

and so on.

240. If we knew the logarithm of 3, this would be the means also of determining a number of other logarithms; as appears from the following examples. Let the \( \log 3 \) be represented by the letter \( y \): then,
\[ \begin{align*}
\log 20 &= y + 1, & \log 300 &= y + 2 \\
\log 3000 &= y + 3, & \log 30000 &= y + 4, & \text{&c.}
\end{align*} \]
\[ \begin{align*}
\log 9 &= 2y, \log 27 &= 3y, & \log 81 &= 4y, & \text{&c. we shall have also,}
\end{align*} \]
\[ \begin{align*}
\log 6 &= x + y, & \log 12 &= 2x + y, & \log 18 &= x + 2y, \\
\log 15 &= \log 3 + \log 5 &= y + 1 - x.
\end{align*} \]

241. We have already seen that all numbers arise from the multiplication of prime numbers. If therefore we only knew the logarithms of all the prime numbers, we could find the logarithms of all the other numbers by simple additions. The number 210, for example, being formed by the factors 2, 3, 5, 7, its logarithm will be \( \log 2 + \log 3 + \log 5 + \log 7 \). In the same manner, since \( 360 = 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5 \), we have \( \log 360 = 3 \log 2 + 2 \log 3 + \log 5 \). It is evident, therefore, that by means of the logarithms of the prime numbers, we may determine those of all others; and that we must first apply to the determination of the former, if we would construct Tables of Logarithms.

CHAPTER XXIII.

Of the Method of expressing Logarithms.

242. We have seen that the logarithm of 2 is greater than \( \frac{1}{2} \), and less than \( \frac{1}{4} \), and that, consequently, the exponent of 10 must fall between those two fractions, in order that
the power may become 2. Now, although we know this, yet whatever fraction we assume on this condition, the power resulting from it will be always an irrational number, greater or less than 2; and, consequently, the logarithm of 2 cannot be accurately expressed by such a fraction: therefore we must content ourselves with determining the value of that logarithm by such an approximation as may render the error of little or no importance; for which purpose, we employ what are called \textit{decimal fractions}, the nature and properties of which ought to be explained as clearly as possible.

243. It is well known that, in the ordinary way of writing numbers by means of the ten figures, or characters,

\[0, 1, 2, 3, 4, 5, 6, 7, 8, 9,\]

the first figure on the right alone has its natural signification; that the figures in the second place have ten times the value which they would have had in the first; that the figures in the third place have a hundred times the value; and those in the fourth a thousand times, and so on: so that as they advance towards the left, they acquire a value ten times greater than they had in the preceding rank. Thus, in the number 1765, the figure 5 is in the first place on the right, and is just equal to \(\frac{1}{10}\); in the second place is 6; but this figure, instead of 6, represents \(10 \times 6\), or 60; the figure 7 is in the third place, and represents \(100 \times 7\), or 700; and lastly, the 1, which is in the fourth place, becomes 1000; so that we read the given number thus:

\textit{One thousand, seven hundred, and sixty-five.}

244. As the value of figures becomes always ten times greater as we go from the right towards the left, and as it consequently becomes continually ten times less as we go from the left towards the right; we may, in conformity with this law, advance still farther towards the right, and obtain figures whose value will continue to become ten times less than in the preceding place: but it must be observed, that the place where the figures have their natural value is marked by a point. So that if we meet, for example, with the number 36.54892, it is to be understood in this manner: the figure 6, in the first place, has its natural value; and the figure 3, which is in the second place to the left, means 30. But the figure 5, which comes after the point, expresses only \(\frac{5}{10}\); and the 4 is equal only to \(\frac{4}{100}\); the figure 8 is equal to \(\frac{8}{1000}\); the figure 9 is equal to \(\frac{9}{10000}\); and the figure 2 is equal to \(\frac{2}{100000}\). We see then, that the more those figures advance towards the right, the more their