Numerical Simulation of Tidal Dispersion Around a Coastal Headland
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ABSTRACT
Tidal flows around headlands can exhibit strong spatial gradients in the Eulerian currents, resulting in complex Lagrangian trajectories and dispersion of the vertically integrated flow. This typically occurs when the horizontal length scale of the headland is comparable to or smaller than the tidal excursion. The effects of these headlands on dispersion are investigated using a depth-averaged hydrodynamic model combined with a particle tracking model. The dispersion of patches of fluid is found to vary by more than an order of magnitude, depending both on position and tidal phase at the time of release. This is due to the infrequent interaction of material with the strongly sheared flow at the tip of the headland, where flow separation occurs during times of maximum tidal flow. Spreading of these patches over many tidal cycles is not Gaussian, but rather shows a patchy, streaky structure.

I. Introduction
In many shallow estuaries and embayments, tidal dispersion plays an important role in the horizontal transport of pollutants, larvae and other suspended or dissolved material. Zimmerman (1976) found that in the Dutch Wadden Sea, the large effective diffusivities calculated from salinity observations (100-1000 m²/s) could be explained by tide-induced dispersion based on an analytic model which considered interactions of a purely oscillatory tidal flow with a spatially varying tide-induced residual eddy field. Awaji et al. (1980) and Awaji (1982) conducted numerical experiments of mixing around a tidally dominated strait and found that even without considering the tide-induced residual effects, the rapid spatial variation in the oscillatory current could give rise to large residual particle displacements and to large values of dispersion (800 m²/s) for material released in the vicinity of the strait.

Insight into this type of dispersion was obtained by Aref (1984), who showed that a completely deterministic, periodic flow of simple form could give rise to rapid mixing due to chaotic particle trajectories. The efficiency of the mixing is controlled by the strength of a small region of high strain and the frequency with which material interacts with this region of high strain. Building on the work of Aref (1984), Pasmanter (1988) examined particle trajectories in a kinematic tidal flow consisting of a mean and oscillatory current that vary harmonically in space. For some values of the flow parameters, he found that the area of released patches grows linearly with time, directly analogous to Brownian motion or turbulent diffusion. For other values, however, Pasmanter (1988) found "islands" of unmixed material embedded in the well-mixed regions, and discovered that the area of patches can grow faster or slower than linearly with time, a phenomenon he termed "anomalous diffusion".

Aref (1984) and Pasmanter (1988) specified simple tidal flow fields, studying the nature of mixing over many tidal cycles, and over a range of flow parameters.
Awaji et al. (1980) and Awaji (1982) used a numerical model to generate a flow field, and primarily studied the nature of mixing over a single tidal cycle. In this paper, the technique of Awaji et al. (1980) is used to investigate the tidal dispersion around an idealized headland, but the simulations are extended over multiple tidal cycles to better understand the longer-term mixing behavior. In the case presented here, the tidal excursion is comparable to the scale of the headland, and the dynamics around the headland are strongly nonlinear, evidenced by flow separation and transient eddy formation. The objectives are to first give a clear description of the basic mixing mechanism around this headland and then to illustrate the dramatic spatial and temporal variation in dispersion characteristics that result.

II. Methods

The basic physics of how vertically well-mixed material is horizontally dispersed around the headland can be understood by neglecting vertical structure and considering only depth-averaged dynamics (Signell, 1989). In this case, the depth-averaged equations governing Eulerian flow in a homogeneous fluid without wind stress can be expressed as:

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{u} \cdot \nabla \mathbf{u} + f(k \times \mathbf{u}) - g \frac{\mathbf{u} | \mathbf{u} |}{h + \eta} + \nabla \cdot (\mathbf{A} \nabla \mathbf{u}),$$

$$\frac{\partial \eta}{\partial t} + \nabla \cdot [\mathbf{u}(h + \eta)] = 0,$$

where $\mathbf{u}$ denotes the depth averaged velocity vector, $\eta$ the free-surface elevation, $h$ the water depth measured from the at-rest sea surface $z=0$, $f$ the Coriolis parameter, $g$ the gravitational acceleration, $C_D$ the bottom drag coefficient, $\mathbf{A}$ the horizontal eddy viscosity, $\nabla$ the horizontal gradient operator, and $k$ the vertical unit vector acting upward.

The numerical scheme to solve these equations is derived from Flather and Heaps (1975), but modified for a curvilinear orthogonal grid (Signell, 1989). The curvilinear grid was chosen to allow increased resolution near the tip of the headland and give a smooth representation of the coastline. The scheme is second-order explicit, using a forward-backward scheme for the gravity waves (e.g., Mesinger and Arakawa, 1976) and an angled-derivative scheme (Roberts and Weiss, 1966) for the advective terms. Spatial variations close to the scale of the grid (2-4 gridlengths) which are treated poorly by the numerical scheme are removed by Shapiro filtering (Shapiro, 1975).

Figure 1 illustrates the geometry of the case study. The channel is 50 km long and 15 km wide and is discretized with a 60 x 30 grid. With the curvilinear grid, this results in a minimum distance between grid points of 53 m at the tip of the headland. Experimentation with grid resolution showed that dispersion results obtained with this grid were negligibly different than results obtained with a grid of twice the resolution. The headland is represented by a Gaussian profile

$$y(x) = A \exp \left[ - \frac{1}{2} \left( \frac{x}{\sigma} \right)^2 \right],$$

with amplitude $A = 7$ km and width specified by $\sigma = 2$ km. The depth increases linearly northward away from the southern boundary, reaching a depth of 20 m at a distance of 3 km from the boundary. The basin is a constant depth of 20 m elsewhere. The western boundary is forced by a normal depth-averaged velocity that varied sinusoidally with an amplitude of 0.5 m/s and a $M_2$ tidal period of 12.42 hours (12 lunar hours). The eastern boundary has a gravity wave radiation condition, the southern boundary has a no-slip condition, and the northern boundary has a free-slip condition. Parameters values are $C_D = 0.0025$, $A_I = 1.0$ m²/s and $f = 1.0 \times 10^{-4}$/s. The specifications of this problem correspond roughly to Gay Head, Massachusetts, a headland at which the detailed spatial structure of the tidal flow has been mapped with an acoustic Doppler current profiler (Geyer and Signell, 1989).

With the explicit solution technique and fine grid spacing, the time step was restricted by the gravity wave Courant condition.