0. INTRODUCTION

The theory of finite simple groups was for a long time a rather isolated and unusual branch of mathematics. It achieved its goal in 1981 when a proof of the classification theorem was completed. This unique proof, comprising several thousand pages of published articles and preprints, leads to the following list of finite simple groups (see [12] for the details): the groups of Lie type, the alternating groups and the 26 sporadic groups. While each of the first two classes has a uniform description, the groups in the third class still have quite different constructions. The largest sporadic group, called the Monster and denoted $F_1$, was predicted independently by B. Fischer and R. Griess in 1973. It contains 20 or 21 of the sporadic groups and has order $>8 \cdot 10^{53}$. This group gave rise to many mysteries even before its actual appearance, promising deep connections with different areas of mathematics.

Many amazing discoveries about the Fischer-Griess Monster were collected in the highly unusual paper "Monstrous Moonshine" by J. Conway and S. Norton [4]. Most of the discoveries were based on the existence of an irreducible $F_1$-module of dimension 196883. B. Fischer, D. Livingstone and M. Thorne computed the entire character table using this assumption, thus providing the numbers for the "Monstrous game". This started with J. McKay's observation that the modular function $j$, in its expansion

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\[
j(q) = q^{-1} + 744 + 196884q + 21493760q^2 + \ldots = \sum_{n \geq 1} a_n q^n,
\]
has a coefficient 196884, which exceeds by only 1 the dimension of the minimal conjectured nontrivial \( F_1 \)-module. Then Thompson observed [24] that the first few coefficients \( a_n \) (for \( n \neq 0 \)) are also simple linear combinations of the dimensions \( d_n \) of irreducible representations of the Monster, e.g., \( a_{-1} = d_1, a_1 = d_1 + d_2, a_2 = d_1 + d_2 + d_3, a_3 = 2d_1 + 2d_2 + d_3 + d_4, \) etc. The coefficient 744 is inessential for the modular property, so one can consider instead the normalized modular function

\[
J(q) = j(q) - 744.
\]

Thompson also proposed replacing the coefficients \( a_n \) in the \( q \)-series for \( J \) by the representations \( V_n \) of \( F_1 \) that they suggested and considering the series

\[
T_m = q^{-1} + 0 + \text{tr } m | V_1 q + \text{tr } m | V_2 q^2 + \ldots
\]

for arbitrary elements \( m \) of the Monster (not just \( m = 1 \)). A great deal of evidence concerning these "Thompson series" was then collected and they appeared to be the normalized generators of function fields of genus 0. Conway and Norton wrote down a list of such series -- one for each conjugacy class in \( F_1 \) -- and conjectured that there exist representations \( V_n \) of \( F_1 \) compatible with this list via (0.3). A. O. L. Atkin, P. Fong and S. Smith (see [23]), using a computer, produced overwhelming evidence for this conjecture, supporting the extreme likelihood that there should exist a natural graded module for \( F_1 \) with the \( J \)-function as character.

About a year after these observations, Griess constructed the 196883-dimensional representation of \( F_1 \) ([14], [15]). However, instead of resolving the mysteries of "Monstrous Moonshine" he added a new one: He gave a construction of the Monster as an automorphism group of a peculiar commutative nonassociative algebra, say \( B_0 \), of dimension 196883.