The present contribution is a brief survey of works done in Kyoto for the past 7 years. As such, it contains no new results. Rather, it is intended to cover an outline of the development that might be of some interest to the people working in vertex operators and related areas. Some more details can be found in several review articles [1], [2] as well as in the original papers [3], [4].

1. The story began with the two dimensional Ising model in statistical physics.

Consider a square lattice of size $M \times N$ on the plane. To each site $(i, j)$ of the lattice, there is attached a random variable $\sigma_{ij}$ with values +1 or -1, which one can think of as a "spin" pointing up or down. There are thus $2^{MN}$ possible configurations $\sigma = (\sigma_{ij})_{1 \leq i \leq M, 1 \leq j \leq N} \in \{+1, -1\}^{MN}$. The model is defined by specifying the energy of a given configuration $\sigma$ as

$$E(\sigma) = -E_1 \sum_{i=1}^{M} \sum_{j=1}^{N} \sigma_{ij} \sigma_{i+1,j} - E_2 \sum_{i=1}^{M} \sum_{j=1}^{N} \sigma_{ij} \sigma_{i,j+1}.$$ 

For definiteness about the boundary, one assumes here that the lattice is wound on a torus, so the index $i$ (resp. $j$) is to be read mod $M$ (resp. $N$).
The probability of a given configuration $\sigma$ to occur is then given by

$$p(\sigma) = Z_{MN}^{-1} e^{-E(\sigma)/kT},$$

where $T = \text{temperature}$, $k = \text{Boltzmann's constant}$, and $Z_{MN} = \sum e^{-E(\sigma)/kT}$ (summed over all $\sigma_{11} = \pm 1$, $\sigma_{12} = \pm 1, \ldots$, $\sigma_{MN} = \pm 1$) is the normalization constant to make the total probability = 1.

One of the quantities of physical interest is the spin correlation function. Given two lattice points, say $(m,n)$ and $(0,0)$, the expectation value of the product $\sigma_{mn}\sigma_{00}$

$$\langle\sigma_{mn}\sigma_{00}\rangle = \sum_\sigma \sigma_{mn}\sigma_{00} p(\sigma)$$

is called the **2-point correlation function**. General n-point correlation functions are defined analogously. There is also the notion of "dual" (sometimes called also "order") correlation functions $\langle\mu_{mn}\mu_{00}\rangle$, etc. These are correlation functions of the Ising model on the dual lattice, whose interaction constants $K_i^* = E_i/kT$ ($i=1,2$). Eventually one takes the infinite lattice limit $M,N \to \infty$ of all these quantities, so that they become functions of $(m,n) \in \mathbb{Z}^2$.

A nice structure emerges when one passes further to another limit, the continuum limit; let the lattice spacing $\epsilon$ tend to 0 while