ALGEBRAS, LATTICES AND STRINGS

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Abstract

A unified construction is given of various types of algebras, including finite dimensional Lie algebras, affine Kac-Moody algebras, Lorentzian algebras and extensions of these by Clifford algebras. This is done by considering integral lattices (i.e. ones such that the scalar product between any two points is an integer) and associating to the points of them the square of whose length is 1 or 2, the contour integral of the dual model vertex operator for emitting a "tachyon". If the scalar product is positive definite, the algebra of these quantities associated with the points of length 2 closes, when the momenta are included, to form a finite dimensional Lie algebra. If the scalar product is positive semi definite, this algebra closes to an affine Kac-Moody algebra when the vertex operators for emitting "photons" are added. If the scalar product is Lorentzian, the algebra closes if the vertex operators for all the emitted states in the dual model are added. Special lattices in 10, 18, and 26 dimensional Lorentzian space are discussed and implications of the dual model no ghost theorem for these algebras are mentioned. This framework links many physical ideas, including concepts in magnetic monopole theory and the fermion-boson equivalence as well as the dual model. (Knowledge of dual models is not assumed but familiarity with aspects of the theory of Lie algebras is presumed in the latter part of this paper.)

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1. INTRODUCTION

Many of the ideas occurring in the quantum theory of the relativistic string (or dual model), such as supersymmetry or dimensional reduction, have found applications outside its immediate context. (For reviews of the subject, see the collection of ref. [1], or ref. [2], for example.) Although exciting and important developments are still being made [3], interest in the theory was most intense in the period 1968 to 1974. The objectives of those working on it progressively broadened from the phenomenological description of high energy scattering (resonances and Regge behavior in particular) to the construction of a completely consistent theory of the strong, and possibly other, interactions. The demand for consistency led to the realization that each dual model or string theory should be considered in a particular space-time dimension, 26 for the original model of Veneziano and others and 10 for the theory of Neveu, Schwarz and Ramond, which includes fermions and introduced supersymmetry. Although formulated in spaces of nonphysical dimensions, these theories possess a high level of consistency and contain very rich algebraic structures; for instance, Yang-Mills gauge theory and supergravity appear as "subtheories" by taking suitable limits. One explanation for why these theories moved from the center of interest is that the technical difficulties presented by handling amplitudes with many fermions proved insuperable on the time scale that theoretical physicists usually expect to solve their problems. Nor could help be found in the mathematical literature because mathematicians were only just discovering the sort of structures which have to be exploited in order to gain an economical understanding of string theories. Subsequently the trade has so far been mainly the other way, with mathematicians taking advantage of the constructions made by physicists.

One of the ideas to have proved useful in mathematics is that of vertex operators $U(r,z)$, which are analytic operators functions of momentum $r$ and a complex variable, $z$. They occur naturally in dual models and they, or rather their moments