Generative Theory of Shape

2.1 New Foundations to Geometry

In my book, A Generative Theory of Shape (Springer-Verlag, 550 pages), I give New Foundations to Geometry. The present chapter gives a brief summary of the New Foundations because the Process Grammar is based on these foundations.

These New Foundations to Geometry directly oppose the Standard Foundations to Geometry that have existed for almost three thousand years. The central proposal in my New Foundations to Geometry is this:

NEW FOUNDATIONS TO GEOMETRY

Leyton 1992

According to the New Foundations to Geometry:
Shape is equivalent to memory storage.

Therefore, in the New Foundations:
Geometry is the mathematical theory of memory storage,
invented by the New Foundations.

Let us see how this opposes the Standard Foundations to Geometry. In the Standard Foundations, a geometric object consists of those properties of a figure that do not change under actions. These unchanged properties are called the invariants of the actions. Geometry began with the study of invariance, in the form of Euclid’s concern with congruence, which is really a concern with invariance (properties that do not change). The full invariance program was explicitly defined in the 19th century by Felix Klein who defined geometry as the study of invariants under groups of transformations. Klein’s invariance program became the basis of 20th century mathematics and physics.
My argument is that the problem with invariants is that they are memoryless. That is, if a property is invariant (unchanged) under an action, then one cannot infer, from the property, that the action has taken place. Thus I argue: Invariants cannot act as memory stores. In consequence, I conclude that the Standard Foundations to Geometry are concerned with memorylessness. In fact, since the Standard Foundations try to maximize the discovery of invariants, those foundations essentially try to maximize memorylessness.

A basic argument of mine is this: The Standard Foundations to Geometry are inappropriate for the computational age, because the computational age is based on memory storage, archival systems, etc., whereas the Standard Foundations are based on the invariants program, which tries to maximize memorylessness.

As a consequence, I embarked on a 30-year project to build up entirely New Foundations to Geometry. Rather than basing geometry on the maximization of memorylessness (the aim of the Standard Foundations), I base geometry on the maximization of memory storage. The result is a theoretical system that is profoundly different, both on a conceptual level and on a detailed mathematical level. Everything in the Standard Foundations is inverted in the New Foundations. For example, whereas in the Standard Foundations, groups are used to describe symmetries, in the New Foundations, groups are used to describe asymmetries. Again, whereas in the Standard Foundations, the goal is reference-frame independence, in the New Foundations, the goal is reference-frame dependence. Again, the relationship between hierarchical levels in the Standard Foundations, e.g., Euclidean to Affine to Projective, are entirely the opposite relationships in the New Foundations. These are all consequences of the fact that the Standard Foundations try to maximize memorylessness, and the New Foundations try to maximize memory storage.

The conceptual structure of the New Foundations is elaborated in my book *Symmetry, Causality, Mind* (MIT Press, 630 pages); and the mathematical structure is elaborated in my book *A Generative Theory of Shape* (Springer-Verlag, 550 pages). The latter book also gives extensive applications of the mathematics to computer-aided design, software engineering, computer vision, gestalt psychology, robotics, music, architecture, and physics. Besides these applications, parts of my mathematical theory have been applied by scientists in over 40 disciplines, such as chemical engineering, meteorology, radiology, geology, botany, structural engineering, mathematical control theory, etc.

The remainder of the present chapter will briefly describe some of the basic concepts and mathematical structures of my New Foundations to Geometry. This will give some useful background to the Process-Grammar.

First, being a generative theory of shape, the New Foundations define any shape by a sequence of operations needed to create it. Next, the New Foundations require that this sequence be intelligent. In fact:

The New Foundations to Geometry elaborate a mathematical theory of intelligence, and base the entire New Foundations on this mathematical theory.

The two most basic principles of this mathematical theory of intelligence are: