Chapter 1
Vectors and Vector Fields

The purpose of this book is to explain in a rigorous way Stokes’s theorem and to facilitate the student’s use of this theorem in applications. Neither of these aims can be achieved without first agreeing on the notation and necessary background concepts of vector calculus, and therein lies the motivation for our introductory chapter.

In the first section we study three operations involving vectors: the dot product of two vectors of $\mathbb{R}^n$, the cross product of two vectors of $\mathbb{R}^3$, and the triple scalar product of three vectors of $\mathbb{R}^3$. These operations have interesting physical and geometric interpretations. For instance, the dot product will be essential in the definition of the line integral (Definition 2.2.1) or work done by a force field in moving a particle along a path. The length of the cross product of two vectors represents the area of the parallelogram spanned by the two vectors, and the triple scalar product of three vectors allows us to evaluate the volume of the parallelepiped that they span, and it plays an important role in calculating the flux of a vector field across a given surface, as we shall see in Chap. 4.