A FULLY ADAPTIVE MULTIRESOLUTION SCHEME FOR SHOCK COMPUTATIONS

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**Abstract.** The scheme is based on Ami Harten's ideas (Harten, 1994), the main tools coming from wavelet theory, in the framework of multiresolution analysis for cell averages. But instead of evolving cell averages on the finest uniform level, we propose to evolve just the cell averages on the grid determined by the significant wavelet coefficients. Typically, there are few cells in each time step, big cells on smooth regions, and smaller ones close to irregularities of the solution. For the numerical flux, we use a simple uniform central finite difference scheme, adapted to the size of each cell. If any of the required neighboring cell averages is not present, it is interpolated from coarser scales. But we switch to ENO scheme in the finest part of the grids. To show the feasibility and efficiency of the method, it is applied to a system arising in polymer-flooding of an oil reservoir. In terms of CPU time and memory requirements, it outperforms Harten's multiresolution algorithm.

The proposed method applies to systems of conservation laws in 1D

\[ \partial_t u(x, t) + \partial_x f(u(x, t)) = 0, \quad u(x, t) \in \mathbb{R}^m. \]  

_Godunov Methods: Theory and Applications_  
In the spirit of finite volume methods, we shall consider the explicit scheme

\[ v_{\mu}^{n+1} = v_{\mu}^{n} - \frac{\Delta t}{h_{\mu}} (f_{\mu} - f_{\mu}^{-}) = [Dv^n]_\mu, \]

where \( \mu \) is a point of an irregular grid \( \Gamma \), \( \mu^- \) is the left neighbor of \( \mu \) in \( \Gamma \), \( v_{\mu}^{n} \approx \frac{1}{\mu - \mu^-} \int_{\mu^-}^{\mu} u(x,t_n)dx \) are approximated cell averages of the solution, \( f_{\mu} = f_{\mu}(v^n) \) are the numerical fluxes, and \( D \) is the numerical evolution operator of the scheme.

According to the definition of \( f_{\mu} \), several schemes of this type have been proposed and successfully applied (LeVeque, 1990). Godunov, Lax-Wendroff, and ENO are some of the popular names. Godunov scheme resolves well the shocks, but accuracy (of first order) is poor in smooth regions. Lax-Wendroff is of second order, but produces dangerous oscillations close to shocks. ENO schemes are good alternatives, with high order and without serious oscillations. But the price is high computational cost.

Ami Harten proposed in (Harten, 1994) a simple strategy to save expensive ENO flux calculations. The basic tools come from multiresolution analysis for cell averages on uniform grids, and the principle is that wavelet coefficients can be used for the characterization of local smoothness. Typically, only few wavelet coefficients are significant. At the finest level, they indicate discontinuity points, where ENO numerical fluxes are computed exactly. Elsewhere, cheaper fluxes can be safely used, or just interpolated from coarser scales. Different applications of this principle have been explored by several authors, see for example (G-Müller and Müller, 1998).

Our scheme also uses Ami Harten’s ideas. But instead of evolving the cell averages on the finest uniform level, we propose to evolve the cell averages on sparse grids associated with the significant wavelet coefficients. This means that the total number of cells is small, with big cells in smooth regions and smaller ones close to irregularities. This task requires improved new tools, which are described next.

A. About the Grids: We shall work with embedded grids \( \Gamma^l \subset \Gamma^{l+1} \). In the uniform setting, \( \Gamma^l = X^l \) are dyadic grids of the unit interval \([0,1]\). In the non uniform setting, \( \Gamma^0 = X^0 \), and for \( l > 0 \), \( \Gamma^l \subset X^l \) is constructed by adding to \( \Gamma^{l-1} \) some points from \( X^l \setminus X^{l-1} \). \( \Lambda^{l-1} = \Gamma^l \setminus \Gamma^{l-1} \) is the set of these new points.

B. Multiresolution Analysis (MRA): In a multilevel setting, the relations between the discrete informations \( g^{l+1} \) and \( g^l \) at two consecutive levels, and the difference of information \( d^l \) between them, are crucial. They lead to multilevel transformations \( g^L \overset{\mathcal{R}^L}{\rightarrow} (g^0, d^0, \ldots, d^{L-1}) \). In wavelet analysis,