Chapter 4

COOPERATIVE GAMES IN A STOCHASTIC ENVIRONMENT

BRUNO APOLLONI¹, SIMONE BASSIS¹, SABRINA GAITO², AND DARIO MALCHIODI¹

¹ – Dipartimento di Scienze dell’Informazione, Università degli Studi di Milano
² – Dipartimento di Matematica “F. Enriques”, Università degli Studi di Milano

4.1 INTRODUCTION

In this chapter we face the task of learning to win a game characterized by highly complex aspects. Our player, Bob, has no information about the ability of his opponent, Nature; moreover the effects of his actions on the outcome of the single contest are neither known nor predictable, since the game mechanism is quite complex and uncertain. Bob can base his strategy only on a monotonicity property: the more Bob increases the value of a given parameter of the game (the one representing his strength), the more his winning ability improves. This class of games exploits the results of game theory [Nash, 1951] only partially since the payoffs associated to each strategy are random variables with unknown distribution. A similar lack of information can be overcome by considering this problem as a dynamic process in a computational learning context. In particular, we refer to the online learning paradigm [Angluin, 1988, Ben-David et al., 1997, Blum, 1996, Littlestone, 1988] along with the statistical inferential framework called algorithmic inference introduced in this book [Apolloni et al., 2002a]. In this framework we evaluate the actions through the construction of confidence intervals for the losing probability, thus abandoning the common goal of optimizing the expected value of a utility function [Blackwell and Girshick, 1979].

From Synapses to Rules, Edited by Apolloni and Kurfess
Kluwer Academic/Plenum Publishers, New York, 2002 75
This game model works well for a wide range of real life situations, from the sphere of economy and finance to the field of biology. We can consider, for example, two firms that produce the same kind of goods. The goal of each firm is to produce a final product qualitatively similar to the competitor's, but with a lesser expenditure of resources. We can also think in terms of an antiviral action against an unidentified virus: we do not know the effect of the application of a certain quantity of antibiotic, since the virus's characteristics are often unclear and the environment in which it survives very complex. We are only sure of the monotonicity property: the more antibiotic we use, the more we can counter the virus; but for obvious reasons we cannot abuse this drug.

The chapter is organized as follows. In Section 2 we describe the game. In Section 3 we solve our inference problem in both batch and on-line modality, analyzing an arising indeterminacy phenomenon that we relieve through a better organization of the available knowledge. In Section 4 we extend the problem to a team game framework and consider an approximately optimal distribution of resources among the players.

4.2 STATEMENTS OF THE GAME

Bob and Nature play the following game.

**Game.** The game consists in a series of contrasts between Bob (B) and Nature (N). In the single contest both players draw randomly an NP-hard problem's instance s from a set Σ (huge, but finite and discrete) and compute an approximate solution, whose accuracy depends on the amounts of resources γ_B and γ_N they employ (let us call them strengths), according to a monotone yet unknown relation.

B wins, ties or loses if his solution is better, equal or worse than the opponent's respectively. B's goal is to find (learn) the minimal strength that will let him lose with at most ε probability and good confidence level δ, rather than win in any future instance. The game is asymmetric: while B can arbitrarily modify his strength, N maintains her value fixed at an initial level.

The complexity of the environment is modeled by choosing an NP-hard [Sahni, 1972] optimization problem. In particular, we have adopted a knapsack problem [Martello and Toth, 1979] as a prototype of this complexity class that reads as follows: given (i) a set of objects, each characterized by a weight and a comfort, and (ii) a capacity to be filled by the weights, find within the subsets of objects fitting the capacity (the