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## **Transitive Multilateral Comparisons of Agricultural Output, Input, and Productivity: A Nonparametric Approach**

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### **ABSTRACT**

EKS Index number methods are examined for multilateral comparisons of input, output, and productivity. The EKS index number procedure was introduced by Caves, Christensen, and Diewert (1982a,b) and is now routinely applied in multilateral comparisons. The EKS method is currently used by Ball et al. (1999, 2000) in the construction of transitive multilateral output and input index numbers. These numbers, in turn, are used in measuring TFP growth in different states. One of the problems associated with using the EKS index is that it considers all binary comparisons to be equally reliable. However, in the context of interstate comparisons, some binary comparisons are intrinsically more reliable than others. The reliability issue arises because of the different output and input structures in different states. This study explores the use of Minimum Spanning Trees (MST) introduced by Hill (1999) in constructing output and productivity comparisons, and also describes a method for generalizing EKS indices. This approach, first proposed by Rao and Timmer (2000), assigns weights to different binary comparisons. The empirical part of this chapter focuses on the implementation of these two methods using inter-state data. The chapter also highlights the problems associated with the application of the MST approach. Empirical results reported and analytical arguments suggest that the weighted EKS method is likely to provide an improved method for constructing transitive multilateral index numbers of output and productivity.

### **INTRODUCTION**

A number of studies have examined the sources of growth in the U.S. farm sector (Ball, 1985; Jorgenson, Gollop, and Fraumeni, 1987; Jorgenson and

Gollop, 1992; Ball et al., 1997). Though their methods differ in important ways, these studies share a common conclusion. Productivity growth was the single most important source of economic growth in post-war agriculture.

More recently, Ball et al. (1999, 2000) focus on agricultural productivity growth at both sector and state levels. A model that accounts for interstate transactions in farm goods links sector-wide and state-specific measures of total factor productivity growth. An interesting conclusion is that the smooth, persistently positive trend typically observed for farm sector productivity growth masks considerable variation across states and regions. The results also indicate that farm sector productivity growth is wholly a function of productivity trends in the individual states. Interstate shifts in production activity and resource reallocations have had little impact.

In this chapter, we provide estimates of the relative levels of productivity for the states. The U.S. Department of Agriculture's Economic Research Service (ERS) has developed a set of state farm production accounts. Salient features of the state accounts are well documented in Ball et al. (1999, 2000). Consequently, our focus will be on methodological issues that arise in the context of making transitive multilateral comparisons. In particular, we will examine critically the widely used Eltetö-Köves-Szulc (EKS) index number procedure in the context of interstate productivity comparisons and describe alternative procedures based on the concept of minimum spanning trees and a weighted or generalised EKS method.

The outline of the chapter is as follows. Section 2 briefly sets the notation used in the paper. A few relevant concepts including transitivity and base invariance are explained. This section also briefly reviews various index number procedures commonly used to construct measures of output, input, and productivity. Section 3 describes the EKS method for multilateral comparisons. We also discuss some of the problems associated with the use of the EKS procedure. Section 4 examines the possibility of constructing chained comparisons between states using the Minimum Spanning Tree (MST) method. Section 5 presents an alternative strategy for constructing generalised or weighted EKS indices that take into account the reliability of various pair-wise comparisons. Results from the empirical application of the method are presented in Section 6. The chapter concludes with a few comments in Section 7.

## NOTATION AND PRELIMINARIES

We begin with some basic assumptions and notational conventions. We assume that there are  $M$  firms or states to be compared. We further assume that there are  $N$  outputs produced. Let  $p_{ij}$  and  $q_{ij}$  denote the price and quantity, respectively, of commodity  $i$  produced in state  $j$ . When the comparison is across