Chapter 5

REDUNDANT IMPLEMENTATIONS OF DISCRETE-TIME LINEAR TIME-ININVARIANT DYNAMIC SYSTEMS

1 INTRODUCTION

This chapter discusses fault tolerance in discrete-time linear time-invariant (LTI) dynamic systems [Hadjicostis and Verghese, 1997; Hadjicostis and Verghese, 1999; Hadjicostis, 1999]. It focuses on redundant implementations that reflect the state of the original system into a larger LTI dynamic system in a linearly encoded form. In essence, this chapter restricts attention to discrete-time LTI dynamic systems and linear coding techniques, both of which are rather standard and well-developed topics in system and coding theory respectively. Interestingly enough, the combination of linear dynamics and coding reveals some novel aspects of the problem, as summarized by the characterization of the class of appropriate redundant implementations given in Theorem 5.1. In most of the fault-tolerance schemes discussed, error detection and correction is performed at the end of each time step, although examples of non-concurrent schemes are also presented [Hadjicostis, 2000; Hadjicostis, 2001].

The restriction to LTI dynamic systems allows the development of an explicit mapping to a hardware implementation and an appropriate fault model. More specifically, the hardware implementations of the fault-tolerant systems that are constructed in this chapter are based on a certain class of signal flow graphs (i.e., interconnections of delay, adder and gain elements) which allow each fault in a system component (adder or multiplier) to be modeled as a corruption of a single variable in the state vector.

2 DISCRETE-TIME LTI DYNAMIC SYSTEMS

Linear time-invariant (LTI) dynamic systems are used in digital filter design, system simulation, model-based control, and other applications [Luenberger, 1979; Kailath, 1980; Roberts and Mullis, 1987]. The state evolution and output
of an LTI dynamic system $S$ are given by

$$q_s[t + 1] = Aq_s[t] + Bx[t], \quad (5.1)$$
$$y[t] = Cq_s[t] + Dx[t], \quad (5.2)$$

where $t$ is the discrete-time index, $q_s[t]$ is the $d$-dimensional state vector, $x[t]$ is the $u$-dimensional input vector, $y[t]$ is the $v$-dimensional output vector, and $A, B, C, D$ are constant matrices of appropriate dimensions. All vectors and matrices have real numbers as entries.

Equivalent state-space models (with $d$-dimensional state vector $q_{s'}[t]$ and with the same input and output vectors) can be obtained through similarity transformation as described in [Luenberger, 1979; Kailath, 1980]:

$$q_{s'}[t + 1] = \begin{bmatrix} T^{-1}AT & T^{-1}B \end{bmatrix} q_s[t] + B'x[t],$$

$$y[t] = \begin{bmatrix} C'T & D' \end{bmatrix} q_s[t] + D'x[t],$$

where $T$ is an invertible $d \times d$ matrix such that $q_s[t] = Tq_{s'}[t]$. The initial conditions for the transformed system can be obtained as $q_{s'}[0] = T^{-1}q_s[0]$. Systems related in such a way are known as similar systems.

3 CHARACTERIZATION OF REDUNDANT IMPLEMENTATIONS

A redundant implementation for the LTI dynamic system $S$ [with state evolution as in Eq. (5.1)] is an LTI dynamic system $H$ with dimension $\eta (\eta \equiv d + s, s > 0)$ and state evolution

$$q_h[t + 1] = \mathcal{A}q_h[t] + Bx[t]. \quad (5.3)$$

The initial state $q_h[0]$, and matrices $\mathcal{A}$ and $B$ of the redundant system $H$ are chosen so that there exists an appropriate one-to-one decoding mapping $\ell$, such that during fault-free operation

$$q_s[t] = \ell(q_h[t])$$

for all $t \geq 0$ [Hadjicostis and Verghese, 1997; Hadjicostis and Verghese, 1999; Hadjicostis, 1999]. Note that according to the setup in Section 3 of Chapter 1, $\ell$ is required to be one-to-one and is only defined from the subset of valid states.