Chapter 6

PARTIAL OBSERVATION CASE

In many situations, the guarantor or creditor will need to invest some amount of money or management time in order to obtain the information on the current position of the guaranteed party’s assets. This situation will change the program in two different dimensions: (1) the dynamics of the controlled assets’ value, \( y(t) \), and (2) the costs faced by the guarantor. Because of these auditing costs, the guarantor will be led to observe the assets’ value process, \( y(t) \), at only some optimally selected time instants. Clearly, the programs derived under full observation can no longer be used here. In the following, we approach the optimal impulse control problem in a partial observation context by analogy with the full observation programs. For this, we assume that the guaranteed party has a passive behavior, thus avoiding the need to consider the game-theoretical framework in which to find the general solution to this problem. Also, we will only discuss the solution to the impulse control problem, since for the optimal stopping problem the solution can be deduced from the former, by replacing the control costs with the stopping cost.

1. THE DECISION PROCESS

Ideally, the guarantor or creditor would like to have information on the assets’ value all the time, but because of the auditing costs, the optimal decision at \( t \) may be only one of these:

1. Let the process \( y(t) \) evolve according to its uncontrolled dynamics.
2. Make a costly audit with cost \( c_0(t) \), without imposing new collateral infusion.
3. Impose a control with no auditing, that brings the level of the assets to a new starting value of \( Y(t) \) at time \( t \), at a cost of \( c_1(Y(t), t) \).
According with the previous courses of action, the guarantor’s goal will be to minimize the objective function given by

\[
J = E_x \{ \int_0^T f(y(t), t) dt + \sum_{i=1}^N \left[ \chi_{\gamma_i=1} c_0(\theta_i) \exp(-r\theta_i) + \chi_{\gamma_i=0} c_1(Y_i, \theta_i) \exp(-r\theta_i) \right] \}
\]

with respect to the variables \(\{N, \theta_i, \gamma_i, Y_i\}\), where \(\theta_i\) are the intervention times (i.e. times at which the process is either audited or impulse-controlled), and \(\gamma_i\) is a control variable, taking the values 1 or 0, according to

\[
\gamma_i = \begin{cases} 
1, & \text{if an audit is made at } \theta_i \\
0, & \text{if a control is imposed at } \theta_i
\end{cases}
\]

The dynamics of the controlled process can be written as

\[
dy(t) = \alpha(y(t), t) dt + \sigma(y(t), t) dz(t) + \sum_{i=1}^N (1 - \gamma_i) \delta(t - \theta_i) \left[ Y_i - y(\theta_i^-) \right],
\]

that is, the process evolves naturally except at times in which an impulse is applied (\(\gamma_i = 0\)). In this case, the impulse size will be \(Y_i - y(\theta_i^-)\), where \(y(\theta_i^-)\) denotes the uncontrolled level of the assets just before \(t = \theta_i\), which may not be known to the guarantor.

In the optimal stopping problem, we want to minimize with respect to the stopping time variable \(\tau\), the functional

\[
J' = E_x \{ \int_0^{T \wedge \tau} f(y(t), t) dt + \sum_{\theta_i < T \wedge \tau} c_0(\theta_i) \exp(-r\theta_i) + g(y(T \wedge \tau), T \wedge \tau) \exp(-r(T \wedge \tau)) \}
\]

where the stopping cost \(g(y(T \wedge \tau), T \wedge \tau)\) replaces the control costs, and where \(\{\theta_i\}_i\) are just the auditing times.

According to Bellman’s principle of optimality, the minimization of \(J\) above amounts at determining \(\forall x, t\) the function