1. INTRODUCTION

Almost 30 years ago, Zadeh (1965) introduced the concept of fuzzy sets. In ordinary set theory, an element is either a member or not a member of a set (in logic, it is indicated by either 1 or 0). In the fuzzy set, whether or not an element belongs to a set may be expressed not just by 1 or 0, but by any value between 0 and 1, indicating different degrees of the membership of the element. For example, 0 indicates a nonmembership, 0.3 a weak membership, 0.9 a strong membership, and 1.0 crisp membership. Building on Zadeh’s work, Yager (1986) offers the following definition:

A fuzzy set is a generalization of the ideas of an ordinary or crisp set. A fuzzy subset can be seen as a predicate whose truth values are drawn from the unit interval, $I = [0, 1]$ rather than the set $(0, 1)$ as in the case of an ordinary set. Thus fuzzy subset has as its underlying logic a multi-valued logic.

Hence, in the context of probability theory, although the fuzzy set concept has a basis similar to that of the classic probability concept, the former departs somewhat from the latter. Furthermore, the conventional set theory is based primarily on objective information, while fuzzy set theory operates on the basis of the premise of subjective judgment. Thus, probability assessments are often characterized by probabilistic phrases, such as probable, likely, unlikely, about 0.4, and approximately $10^{-2}$, which are often difficult to quantity. They are linguistic probability values assessed by subjective judgment. They reflect the way humans think and express a certainty value in an imprecise but useful way.

The use of these expressions/phrases is abundant in a multitude of disciplines; however, their usefulness is not fully capitalized until the introduction of the fuzzy set concept. In terms of probability, the need for using linguistic phrases for uncertainties emerges when crisp probabilities are not available, or when the event of interest is subjective in nature.


Despite the inevitable applications to structural mechanics, the integration of the fuzzy set concept into this area is relatively new. This may be due to both its subjective nature and its departure from the ordinary set theory. Uncertainties in engineering (or for this matter, in structural mechanics) are no different than those in other areas. Many of these uncertainties have objective characteristics, but many others are established—or more appropriately established—on the basis of subjective judgment.

As an example, in some types of problems the range (lower and upper limit) of, say, a uniform probability distribution function of a variable is often determined more realistically through the use of subjective judgment. As another example, it would be more realistic and convenient for someone to say that the chance of a house in San Francisco being severely damaged by the next earthquake is highly probable rather than, say, 98%. The probability that a house is severely damaged during an earthquake cannot be defined precisely because there is no sharp dividing line between severe damage and no severe damage. Furthermore, objective information concerning the variables used to establish the probability is often not quantifiable. Hence, for cases such as this, for which there is no sharp boundary between failure and no failure and/or for which sufficient quantifiable objective data are not available, a vague response such as highly probable would be more appropriate. Fuzzy set concepts can be used to analyze such cases.

2. NOTATIONS AND ABBREVIATIONS

2.1. Notations

\begin{align*}
A & \quad \text{A fuzzy set representing a linguistic value} \\
\mathcal{A} & \quad \text{A universe of discourse} \\
A \subseteq \mathcal{A} & \quad A \text{ is a subset of } \mathcal{A} \\
A \subseteq \mathcal{A} & \quad A \text{ is a subset or the set of } \mathcal{A} \\
A \supset B & \quad A \text{ implies } B; \text{ if } A \text{ then } B \\
\forall A_j & \quad \text{For all } A_j; j = 1, 2, \ldots \\
\forall a_j & \quad \text{For all } a_j \text{ contained in } A \\
C = A \circ B & \quad C \text{ is the composition of } A \text{ and } B \\
K(j) & \quad \text{Fuzzy kernel} \\
N_0 & \quad \text{A crisp singleton} \\
n & \quad \text{Exponent of a probability} \\
[1][n] & \quad \text{Fuzzy singleton of the exponent } n \\
R_{IP} = I \times P & \quad \text{A fuzzy relation between } I \text{ and } P; \text{ also Cartesian product of } I \text{ and } P \\
R^T & \quad \text{Transpose of fuzzy matrix } R \\
\alpha & \quad \text{Composite fuzzy relation operator} \\
\mu_{\alpha}(a_j) & \quad \text{Membership function of fuzzy set } A
\end{align*}