THE BIG GAME OF ENERGY AND ENTROPY

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PRELIMINARY CONSIDERATIONS

Thinking science for teaching can be tackled from different points of view because of the different problems involved. We can approach it on the psycho-pedagogical level, involving questions like cognition, constructivism, building of knowledge, etc.. This is a general level, somewhat independent of the specific field of knowledge to be treated. Another level is that which we can specifically define as methodological, concerned with how to go about teaching students a specific topic (conceptual maps, texts, videos, lab, organization of subject matter, etc.). In this framework the topic is taken for granted in its standard form, and no questions are raised about its validity.

Last, and most important, is the level at which one challenges the standard point of view, bringing into play the most recent developments in physics in an effort to achieve a different conceptual organization in order to better understand what is being taught.

We consider this to be of utmost importance, especially given the rapid progress in this field. The tendency in physics teaching - also at undergraduate and primary school levels - is to introduce as much "modern physics" as possible, meaning quantum mechanics, relativity, cosmology, etc., as included in the official curriculum. It is our opinion, however, that these are very difficult to grasp if students have only been exposed to "traditional" physics. It would be necessary to re-organize the subject matter, taking the latest findings into account from the beginning. This would be thinking physics for teaching.

This paper is an attempt to move in this direction. We are searching for physical entities taken from as wide a range as possible that can be used as cognitive organizers. Obviously energy would suit the purpose. However energy alone is not enough, unless we limit ourselves to "ideal" physics, comprised of conservative systems only, without considering the thermodynamics that characterize any real phenomenon. The same fundamental property of energy, i.e. its conservation, that derives directly from the epistemological assumption of homogeneity of time, is usually reduced to being considered a trivial consequence of the laws of motion, and is consequently associated almost entirely with mechanics.

This is immediately evident in the Hamiltonian formalism: From the total energy $H(q_i, p_j)$ we obtain the laws of motion:

*Thinking Physics for Teaching*, Edited by Carlo Bernardini et al.
Plenum Press, New York, 1995 269
As a consequence,

\[ \dot{p}_i = -\frac{\partial H}{\partial q_i}, \quad \dot{q}_i = \frac{\partial H}{\partial p_i} \]

As a consequence

\[ \frac{dH}{dt} = \left( \frac{\partial H}{\partial q_i} \right) \dot{q}_i + \left( \frac{\partial H}{\partial p_i} \right) \dot{p}_i = 0. \]

The point is that we need entropy together with energy: time is not only homogeneous, but also unidirectional, and this is an inescapable fact of nature. As a consequence, energy and entropy are closely connected in their epistemological basis, and the game of their mutual interaction can actually furnish a cognitive organizer for almost all of physics.

For this purpose, however, we must overcome our present, limited concept of entropy which is normally employed in elementary thermodynamics and take advantage of recent and past developments that reveal the importance of its connection with other concepts, such as information, correlations, complexity, and so on. We will demonstrate that we can associate a well-defined value of entropy not only to thermodynamics, but also to mechanics, in describing a system, whether it is in an equilibrium state or not.

**HOW MANY ENTROPIES?**

Among the various entities that physics uses for its purposes, entropy is one of the most mysterious in terms of meaning and the extent to which it can be part of a single theory.

Of course chemists use it continuously in calculations, and the numerical values of the specific entropy of various substances are listed in various handbooks. In such cases, however, entropy is nothing other than an empirical quantity evaluated according to some experimental results.

The problem arises when we try to define it from a theoretical standpoint and to calculate its value, once a suitable model has been found.

First we notice that there are many other entities which fit the description. In particular:

1. entropy can be defined in a thermodynamical sense as the extensive variable conjugate to temperature (unusual because not conserved). Here a central role is played by the concept of adiabaticity, i.e., heat. Caratheodory's elaborate treatment is based on the assumption that in the neighbourhood of any given state there are an infinite number of states that are adiabatically inaccessible to it. As a result, there exists an integral denominator for the infinitesimal exchange of heat which permits us to put \( dS = \delta Q/T \).

2. If one accepts the widespread notion that thermodynamics is nothing other than a macroscopic statistical description of a multi-bodied system, entropy becomes a property of an ensemble of systems, rather than of a single system. Actually, the system is assumed to be in a well defined, although unknown, state (a mechanical state) in which entropy is zero. Then entropy appears as a measure of the ensemble of all the microstates which correspond to some given macroscopic constraints. The same thing happens in the quantum expression \( \text{tr} \rho \ln \rho \), \( \rho \) being the density matrix. The true state of the system is always a pure state (that is, a quantum-probabilistic superposition of eigenstates) that evolves reversibly according to Schrödinger's equation, and for which entropy is zero. On the contrary, the density matrix does not correspond to a true state, being the statistical property of an