EQUIVALENCE RELATIONS FOR
STOCHASTIC AUTOMATA
NETWORKS
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ABSTRACT

Stochastic Automata Networks (SANs) are an efficient means to describe and analyze parallel systems under Markovian assumptions. The main advantage of SANs is the possibility to describe and analyze a complex parallel system in a compositional way such that the transition matrix of the Markov chain underlying the complete SAN can be described in a compositional way using only small matrices specifying single automata and combine these matrices by means of tensor operations. This approach allows, up to a certain extent, the handling of the state space explosion resulting from complex Markov models. In this paper equivalence relations for stochastic automata are introduced such that an automaton in a network can be substituted by an equivalent and usually smaller automaton without affecting the results of an analysis. We consider equivalence according to stationary and transient analysis of SANs.

1 INTRODUCTION

Stochastic Automata Networks (SANs) under Markovian timing have been proposed by Plateau and her coworkers in several papers [11, 12] as a good modeling tool for the quantitative analysis of complex parallel and concurrent systems. A SAN is described by a number of isolated stochastic automata (SAs) which are dependent due to synchronization and transition rates of a local SA depending on the state of other SAs in the SAN. The main advantage of SANs over most other approaches to specify Markov processes is that the transition matrix of a Markov process underlying a SAN can be completely described by the combination of small matrices specifying a single SA which are combined using operations from tensor algebra. This decomposition of the transition matrix can be exploited in iterative numerical solution techniques by implementing the
basic operation, namely vector matrix multiplication, without first generating
the complete transition matrix. Thus SANs allow a significant reduction of
storage requirements since the large transition matrix needs not to be stored
as a whole. The approach therefore enables the analysis of very large models
which cannot be handled with conventional analysis techniques.

In this paper we consider a new aspect of the analysis of SANs. The motiva-
tion is to reduce the complexity of a SAN and compute exact results from the
reduced model. The goal is to find for a SA a smaller but equivalently behav-
ing representation. Equivalence is a very important concept in the functional
analysis of complex systems which result from the composition of less complex
parts like it is the case in process algebras (see e.g. [9]). Equivalence according
to quantitative aspects is not so established since quantitative analysis based
on composition was nearly unknown in the past. However, here we show that
a well formalized concept of equivalence according to quantitative results can
be defined for SANs. Apart from theoretical importance equivalence in SANs
is practically applicable. An equivalent SA can be computed for a given SA
using only very limited information about the rest of the SAN, in particular,
only matrices for a single SA need to be considered. However, the use of the
reduced SA yields a reduction of the overall state space by a factor similar to
the reduction factor reached on the local state space of the SA.

The results in this paper are based on lumpability in finite Markov chains
as originally defined in [8], basic results for this paper are given in [3]. For a
different class of models, which are described by hierarchical queuing networks
or stochastic Petri nets, exact aggregation of submodels has been introduced in
[1, 2]. However, although the basic ideas of these papers are similar there are
clear differences to the results presented here since the model class and structure
of the transition matrix is different, in particular, communication between SAs
is synchronous rather than asynchronous. Furthermore equivalence of SAs is
introduced here in a way which is very similar to the inductive definition of
equivalence according to the functional behavior of a process (see e.g., [9]).
This allows the computation of a, up to the ordering of states unique, smallest
representation of a SA according to a given equivalence relation.

The outline of the rest of the paper is as follows. In the next section we
introduce SANs and the structure of the underlying transition matrix based on
the combination of matrices for a single SA. Afterwards equivalence and exact
aggregation of Markov reward processes is considered in general. In section 4
equivalence of SAs in a SAN is introduced followed by two examples for the
approach. The paper ends with the conclusions.