GRAPHS AND STOCHASTIC AUTOMATA NETWORKS
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ABSTRACT

We show how some graph theoretical arguments may be used to reduce the complexity of the computation of the steady-state distribution of Markov chain. We consider the directed graph associated to a Markov chain derived from a Stochastic Automata Network (SAN). The structural properties of the automata are used to establish new various results. First, we establish the complexity of the resolution for Stochastic Automata Networks with a sparse matrix representation of the automata. This results are used to compare simple SAN (i.e. without functions) with methods which generates a sparse representation of Markov chains (i.e. Markovian Petri Nets for instance) on some examples.

Then, we show how to apply state reduction techniques on a chain associated to a SAN. We present an algorithm to solve the steady-state equations and we prove its complexity. Finally, we extend our algorithm to allow the semi-parametric analysis of Stochastic Automata Networks.

1 INTRODUCTION

The complexity of the new generation of distributed systems has made the availability of efficient modeling tools a very important issue. These systems are complex to study because they lead to models with an exponential growth of the number of states. We lack analytical results for systems with complex synchronizations and naive numerical analysis is not sufficient.

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In this paper, we investigate the complexity and the computational aspects of the numerical and parametric resolution of continuous time finite Markov chains. These chains are often large and sparse. They usually lead to a modeling problem (how to build large Markov chains from the specifications of our problem) and to an algorithmic problem (how to solve the steady-state distribution with a sufficient accuracy).

In this paper, we consider an approach designed for the representation of parallel systems: Stochastic Automata Networks (SAN) [12]. We advocate that this methodology answers the first problem (i.e. the design of large Markov chain from specifications) and may help to derive new numerical methods. Indeed, the main advantage of SAN is the automatic generation of the transition matrix using tensor algebra. Thus, structural properties (i.e. properties of the underlying directed graphs) of the chain may be verified without explicitly generating the transition matrix. Directed graphs may be easily associated to Markov chains since Markov chains on discrete space are state-transition diagrams with transition rates. Some graph properties have been recently used to prove analytical results for the steady-state distribution of Markov chains [3, 7].

In this paper, the structural properties of the automata are used to establish new various results. These results are connected to a new formalism, the so-called process algebra method. This formalism has been recently developed by various authors, see [1] for applications to Markov chains and [2] for stochastic Petri nets.

First, we establish the complexity of the iterative resolution for Stochastic Automata Networks with a sparse matrix representation of the automata. More precisely, we obtain the complexity of the product of a vector by a sum of tensor product of matrices. This algorithm is the main part of various iterative resolution scheme such as the power method or Arnoldi's algorithm. We also obtain a bound on the number of non zero elements in the transition matrix. These results are used to compare simple SAN (i.e. without functions) with methods which generates a sparse representation of Markov chains (i.e. Markovian Petri Nets for instance). These results and comparisons were previously made only for complete matrix representation.

Then, we investigate some state reduction techniques to obtain new algorithms for the computation of the steady state distribution of Markov chains. These algorithms may have a smaller complexity for some types of SAN than the first algorithm developed under this formalism [12].

Intuitively, state reduction techniques consists of using a partition of the generator $Q$ of the Markov chain into four blocks. This partition is usually suggested by the problem (see [8] for a bibliography on this topic).