14.1 CELLULAR NETWORK CHIPS WITH ANNEALING CAPABILITY

Local interconnection and simple synaptic operators are the most attractive features of the CNN for VLSI implementation in high-speed, real-time applications. Several hardware implementations of the CNN have been reported in the literatures [1]-[11]. The CMOS VLSI design of a continuous-time shift-invariant CNN with digitally-programmable operators is considered. In addition, the circuits for hardware annealing [12, 13, 14, 15] is included to provide the flexibility of the network in a variety of applications.

14.1.1 System Computing Architecture

The differential equation governing a CNN is rewritten here for convenience

\[
C_{z} \frac{dx}{dt} = -T_{x} x + Ay + Bu + I_{b} w, \tag{14.1}
\]

where \( y \in D^{N} = \{ y \in R^{N} : -1 \leq y \leq 1 ; k = 1, 2, \ldots, N \} \), and \( A \) is an N-by-N real symmetric matrix defined as

\[
A = \begin{bmatrix}
A_{0} & A_{1} & 0 \\
A_{1} & A_{0} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
0 & A_{1} & A_{0}
\end{bmatrix}.
\tag{14.2}
\]
Here \( \mathbf{w} = [1 \cdots 1]^T \) is an \( N \)-by-1 constant vector. In (14.2), \( \mathbf{A}_0 \) and \( \mathbf{A}_1 \) are two \( m \times m \) Toeplitz matrices with elements determined by a given cloning template. The synapse weights of the shift-invariant CNN can be described by the feedback and feedforward cloning templates

\[
\mathbf{T}_A = \begin{bmatrix} a_0 & a_1 & a_2 \\ a_1 & a_0 & a_1 \\ a_2 & a_1 & a_2 \end{bmatrix} \quad \text{and} \quad \mathbf{T}_B = \begin{bmatrix} b_0 & b_1 & b_2 \\ b_1 & b_0 & b_1 \\ b_2 & b_1 & b_2 \end{bmatrix} \tag{14.3}
\]

respectively, where all elements represent the normalized numbers to \( T_x = 1/R_x \), and \( a_0 = A(i,j;i,j)/T_x > 1 \). Then \( \mathbf{A}_0 = \text{toeplitz}(a_0, a_1) \) and \( \mathbf{A}_1 = \text{toeplitz}(a_1, a_2) \), where \( \text{toeplitz}(a, b) \) is defined as the Toeplitz matrix with all \( a \)'s in the main diagonal, and all \( b \)'s in above and below the main diagonal. The input vector \( \mathbf{v}_u \) and the matrix \( \mathbf{B} \) can be defined in a similar way. Note that the matrix \( \mathbf{A} \) or \( \mathbf{B} \) is not Toeplitz, but always real symmetric as long as the cloning templates are shift-invariant. Let \( u, x \) and \( y \) be the input, state and output voltages normalized to \( v_{yij}(t = +\infty) \). Then, the maximum value of \( x \) in the steady state is the sum of the absolute values of all inputs from the neighborhood cells,

\[
|x_{max}| = \left| \sum_{i,j=1}^{3} |\mathbf{T}_A(i,j)| + \sum_{i,j=1}^{3} |\mathbf{T}_B(i,j)| \right| + |x_0| = a_0 + 4(|a_1| + |a_2|) + b_0 + 4(|b_1| + |b_2|) + |x_0|, \tag{14.4}
\]

where \( |u| \leq 1, |y| \leq 1 \), and \( x_0 = I/T_x \). For example, if \( a_0 = 2, a_1 = a_2 = b_0 = 1, \) and \( b_1 = b_2 = 0 \), then \( |x_{max}| = 11 \). The neuron cell should be able to handle the state voltage of the range \( |x| \leq |x_{max}| \).

The shift-invariant architecture of the CNN is the most advantageous feature in realizing the network in electronic circuits using the VLSI technologies. The basic cell consists of a summing circuit for the right-hand side of (14.1) and a nonlinear function generation circuit. The neuron cells and operator weights are the basic elements of the CNN hardware. To accommodate a large-size neural network in a single VLSI chip, it is especially important to design the low-complexity and high-accuracy circuits for the efficient realization of neurons and weights. The shift-invariant property of the template weights in the CNN implies that the number of distinct weights are much smaller than that of neighboring cells. If the symmetry of the weights is assumed except in the edges, the number of the \( r \)-neighborhood cells \( C(k,l) \in N_r(i,j) \) of the cell \( C(i,j), \forall i, j \), is

\[
N_C(r) = 1 + 4r(r + 1), \tag{14.5}
\]