THE HYDRODYNAMIC MODEL FOR HIGH-ENERGY HEAVY ION REACTIONS

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Introduction

Hydrodynamic models\textsuperscript{1-3} have the great advantage of allowing for a simple relativistic formulation and explicit use of the equation of state of excited hadronic matter. On the other hand, when applying one-fluid hydrodynamics to heavy ion collisions, one assumes local thermodynamic equilibrium, which is probably not a good approximation at high bombarding energies. Nevertheless, at relatively low bombarding energies the hydrodynamic model has been successfully used to describe heavy ion collisions\textsuperscript{4-6} and even at CERN energies it seems not to contradict experimental data\textsuperscript{7}. Note that also the Landau model\textsuperscript{8} and related models\textsuperscript{9-12} are successfully applied to describe certain observables of multiparticle production in hadron–hadron, hadron–nucleus and nucleus–nucleus collisions.

In the following we will first examine the lower energy hydrodynamics and in more detail the equation of state and the approach of viscosity in nonrelativistic situations. Then we discuss the relativistic formulation of the hydrodynamic model and introduce a model equation of state with a deconfinement phase transition. Finally we give a short exposition of the fundamental problem of viscous relativistic hydrodynamics.

Lower Energy Hydrodynamics

Equations of Motion

The equations of motion of non-viscous hydrodynamics simply correspond to conservation conditions for the macroscopic fields $\rho$ (density), $\rho \vec{v}$ (momentum density),

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and $\rho E$ (energy density)

\[
\begin{align*}
\partial_t \rho + \nabla \cdot (\rho \vec{v}) &= 0, \\
\partial_t (\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) &= -\nabla p, \\
\partial_t (\rho E) + \nabla \cdot (\rho E \vec{v}) &= -\nabla \cdot (\vec{v} p).
\end{align*}
\]

(1) (2) (3)

Here $\vec{v}$ is the flow velocity, $E$ the energy per particle (containing kinetic and internal energy), while $p$ stands for the pressure, which should be determined from $\rho$ and $E$ using the equation of state. The equation of state being essentially unknown and heuristic functions were usually employed.

**Typical Heuristic Equations of State**

A widely used but in principle unjustified assumption is that of separability of the equation of state into a compressional and a thermal part. Formally the internal energy $W(\rho, T)$ enters into the total energy per nucleon via

\[ E = \frac{1}{2} m v^2 + W(\rho, T), \]

and the latter is assumed to split according to

\[ W(\rho, T) = W_0(\rho) + W_{\text{th}}(\rho, T) \]

with $W_0(\rho) = W(\rho, T = 0)$. This function is often simply referred to as the equation of state, because it contains the unknown compressional behaviour of nuclear matter, while $W_{\text{th}}$ is usually assumed to be given by the Fermi gas expressions.

The known properties of nuclear matter impose some constraints on the function $W_0(\rho)$. The equilibrium value must be $W_0(\rho_0) = -B_0$ and the incompressibility should be $K = (9\partial^2 W_0/\partial \rho^2)_{\rho_0}$. In principle vacuum properties should also be reproduced, i.e. $W_0(\rho = 0) = 0$ with a quadratic rise for low densities, where nuclear matter should be close to a gas of noninteracting nucleons. In reality, however, nuclear matter is unstable in this region with respect to a breakup into nucleons and light nuclei, so that the equation of state is not required to describe this region correctly. In practical calculations the behaviour of nuclear matter in the breakup region is described by different models, so that the functional form of $W_0(\rho)$ does not matter there. The breakup density is expected to be between one third and one half of $\rho_0$.

Another condition is provided by causality. Relativistically, the speed of sound is given by the formula

\[
c_s^2 = \left. \frac{\partial W(\rho, s)}{\partial \epsilon} \right|_s,
\]

which in the case of zero entropy reduces to $c_s^2 = dW_0(\rho)/d\epsilon$, where $\epsilon$ is the total internal energy density including the rest mass, $\epsilon = \rho (mc^2 + W)$. Now $c_s$ should not exceed the speed of light. This leads to the condition that $W_0(\rho)$ should not rise more rapidly than linearly with $\rho$ for large $\rho$. However, since $c_s^2$ is related to the curvature of $W_0(\rho)$, there may also be problems if this function contains regions of larger curvature.

Widely used parametrizations are (to abbreviate the expressions we use the compression ratio $x = \rho/\rho_0$):