Chapter VI

DETERMINATION OF RESTING POTENTIAL

A. Modified Nernst Equation

The basic Nernst equation has been modified in several ways for special situations. For example, in the concentration cell illustrated in Chapter V (Fig. V-1), a cell in which a single salt (both ions of same valence) is distributed across the membrane at two different concentrations, the PD developed across the membrane \( E_m \) can be calculated from the following equation (of course, this equation does not apply to biological, or living, cells in which several salts are distributed across the cell membrane)

\[
E_m = \frac{U_c - U_a}{U_c + U_a} \frac{-61 \text{ mV}}{z} \log \frac{[\text{salt}]_1}{[\text{salt}]_2}
\]

where \([\text{salt}]_1\), and \([\text{salt}]_2\) are the concentrations of the salt on side 1 and side 2, and \(U_c\) and \(U_a\) are the mobilities of the cations and anions, respectively, through the membrane. Thus, when the mobility (or permeability) of the cation and anion are equal \((U_c = U_a)\), \(E_m\) is zero, regardless of the equilibrium potentials. When the anion is impermeable \((U_a = 0)\), then the mobility fraction in Equation 1 becomes 1.0, and the equation reduces to the simple Nernst equation. When the cation is impermeable \((U_c = 0)\), the fraction becomes -1, and the same numerical value of \(E_m\) is produced, but of the opposite sign. Equation 1 can be used to calculate \(E_m\) for any combination of \(U_a\) and \(U_c\). For example, if \(U_a = 0.5\ U_c\), then for the problem illustrated in Chapter V (Fig. V-1), \(E_m\) is about -20 mV (side 1 negative). Thus, Equation 1 relates the membrane potential to the relative mobilities.

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B. Constant Field Equation

An important modification of the Nernst equation in common use for calculating the membrane potential, or more importantly for determining the \( P_{Na} / P_{K} \) ratio, is the Goldman-Hodgkin-Katz (Goldman, 1943; Hodgkin & Katz, 1949) constant field equation

\[
E_m = \frac{-RT}{F} \ln \frac{P_K [K^+]_i + P_{Na} [Na^+]_i + P_{Cl} [Cl^-]_o}{P_K [K^+]_o + P_{Na} [Na^+]_o + P_{Cl} [Cl^-]_i}
\]

eq 2

where \( P_K, P_{Na}, \) and \( P_{Cl} \) are the membrane permeabilities for \( K^+, Na^+, \) and \( Cl^- \), respectively. The \( P_K [K]_i \) product, for example, is given in the units of a flux (mol/sec per cm²), but the entire right-hand term (fraction) is dimensionless. Plugging in the numerical values for the constants and converting from natural logarithm, or \( \ln \), to logarithm to the base 10, or \( \log (\ln N = 2.3 \log N) \), and dividing the right-hand term by \( P_K \) gives

\[
E_m = -61 mV \log \frac{[K^+]_i + \frac{P_{Na}}{P_K} [Na^+]_i + \frac{P_{Cl}}{P_K} [Cl^-]_o}{[K^+]_o + \frac{P_{Na}}{P_K} [Na^+]_o + \frac{P_{Cl}}{P_K} [Cl^-]_i}
\]

eq 2a

\( E_m \) expressed in mV. Again, the right-hand term (fraction) is dimensionless.

Any ion that is passively distributed, i.e., not actively pumped, however, cannot determine the resting potential, because the ion's distribution must follow the resting potential. Therefore, when \( Cl^- \) is passively distributed, it drops out of consideration. (This is not to say that transient net movements of \( Cl^- \) inward or outward do not influence \( E_m \), because they