A NEW GRAPH BASED PRIME COMPUTATION TECHNIQUE

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ABSTRACT

Computing prime and essential primes of Boolean functions is a problem that has applications in many different areas of computer science including computer design [2, 9], automated reasoning [12], and reliability analysis [8]. Though much effort has been spent on this problem over the last decades, all the prime computation techniques that have been developed so far are of limited power because they all manipulate sets of primes explicitly. This chapter presents a new prime computation procedure that overcomes this limitation because its complexity is not related to the number of primes to be computed but to the sizes of the graphs used to represent the sets of primes implicitly.

2.1 INTRODUCTION

A prime implicant computation technique has been recently introduced that makes possible to handle Boolean functions with sets of primes and of essential primes too large to be explicitly built [6, 7]. The key ideas that underlie this technique are to represent and to compute these sets implicitly using metaproduc ts [6] that are a canonical representation of sets of products, and to represent these metaproducts with binary decision diagrams (BDD) [4]. This technique overcomes the limitations of all previously known prime computation techniques because its cost is related to the size of the BDDs it manipulates, and not to the number of primes to be computed.
The BDDs of metaproducts that represent sets of primes and of essential primes are very redundant. The elimination of these redundancies produces dramatically smaller BDDs, and it can be done in such a way that the resulting representation, called the Implicit Prime Set (IPS) representation, is still canonical. However, metaproducts were defined in such a way that set operations on set products correspond to logical operations on metaproducts, and this correspondence does not exist anymore with IPSs, so that the prime computation techniques presented in [7] cannot be implemented using IPSs because they manipulate sets of products that are not all primes and such sets cannot be represented with IPSs.

This chapter presents the implicit prime set representation and the new prime and essential prime computation procedure based on this representation. This procedure has been shown by experience to be more powerful than any previously known procedure, including the one based on metaproducts, since it can handle with success all the vectorial Boolean functions described in the MCNC benchmark [15] that include examples that had never been treated before.

This chapter is divided in 6 parts. Section 2.2 presents the problems addressed here, and introduces the notations and the elementary concepts that will be used to solve them. We assume the reader familiar with binary decision diagrams [4]. Section 2.3 briefly presents the metaproduct representation, explains why BDDs of metaproducts of prime sets are redundant, and then introduces the IPS representation. Section 2.4 presents the IPS based prime and essential prime computation of Boolean functions. Section 2.5 presents the theorems that allow us to handle vectorial Boolean functions using the procedure presented here. Section 2.6 gives experimental results obtained with this new procedure.

2.2 DEFINITIONS AND NOTATIONS

2.2.1 Formulas and Functions

A propositional formula built out of \( n \) propositional variables denotes a unique Boolean function from \( \{0,1\}^n \) into \( \{0,1\} \) [10]. A literal is a propositional variable \( x_k \) or its negation, also noted \( \overline{x_k} \). We note \( \Delta(x_k, L, H) \) the function \((\overline{x_k} \land L) \lor (x_k \land H)\). We note \((f_{\overline{x_k}}, f_{x_k})\) the unique couple of functions obtained using the Shannon expansion of \( f \) with respect to \( x_k \) [1],

A function \( f \) from a set \( E \) into \( \{0,1\} \) denotes a unique subset of \( E \) that is