In this chapter, theories for deadend microfiltration are presented. The first part of the chapter focuses on the sieving or surface filtration mechanism, which is dominant when the particles are physically too large to pass through the pores of the filter medium. The primary goal is to predict the flux decline due to the buildup of the rejected particles on the membrane surface. We start with Darcy’s law for the relationship between flux and pressure drop across a cake layer and membrane in series. This relationship is used to describe transient cake buildup and flux decline for batch operation of deadend microfilters. The analysis is then extended to continuous operation of rotary drum vacuum filters.

The second part of this chapter focuses on the various depth filtration mechanisms that occur when smaller particles are able to enter the interior of the membrane pores. The primary goal is to predict the efficiency of a given filter in removing particles of a given size. Particle capture by depth filtration from both gases and liquids is reviewed. In general, particles are more easily removed by depth filtration mechanisms from gases than from liquids.
DARCY'S LAW

When the sieving mechanism of microfiltration is dominant, a cake layer of rejected particles usually forms on the membrane surface, as shown in Figure 32-1. The pressure-driven permeate flux through this cake layer and the membrane may be described by Darcy's law:

\[
J \equiv \frac{1}{A} \frac{dV}{dt} = \frac{\Delta p}{\eta_0 (R_m + R_c)}, \quad (32-1)
\]

where \( J \) is the permeate or volumetric flux and denoted commonly as \( J_v \) in the membrane literature, \( V \) is the total volume of permeate, \( A \) is the membrane area, \( t \) is the filtration time, \( \Delta p \) is the pressure drop imposed across the cake and membrane, \( \eta_0 \) is the viscosity of the suspending fluid, \( R_m \) is the membrane resistance (which can increase with time due to membrane fouling and compaction), and \( R_c \) is the cake resistance (which can increase with time due to cake buildup and compression). If this equation is to be useful, knowledge regarding the membrane and cake resistances is needed. Although such knowledge is best gained from experimental measurements, we present semi-empirical formulas that may be used to estimate \( R_m \) and \( R_c \).

Membrane Resistance

Membrane resistance clearly depends on the membrane thickness, its nominal pore size, and various morphological features such as the tortuosity, porosity, and pore size distribution. For a membrane whose pores consist of cylindrical capillaries of uniform radius perpendicular to the face of the membrane, the resistance can easily be calculated. Using the Hagen-Poiseuille equation, the flux through such a membrane is

\[
J = \frac{n_p \pi r_p^4 \Delta p_m}{8 \eta_0 l}, \quad (32-2)
\]

where \( n_p \) is the number of pores per unit area, \( r_p \) is the pore radius, \( l \) is the membrane thickness, and \( \Delta p_m \) is the transmembrane pressure drop. From this, the membrane resistance is given by

\[
R_m = \frac{\Delta p_m}{\eta_0 J} = \frac{8l}{n_p \pi r_p^4}, \quad (32-3)
\]

indicating that the membrane resistance increases with increasing membrane thickness and decreases with increasing pore size and number density. It may also increase with time if fouling or particle capture in the membrane interior occurs.

It is often convenient to define the porosity, \( \epsilon_m = \text{membrane void volume}/\text{total volume} \), and the specific surface area, \( S_m = \text{porous surface area}/\text{solids volume} \). For a membrane with uniform cylindrical pores, it is easy to show that \( \epsilon_m = n_p \pi r_p^2 \) and \( S_m = 2\pi n_p r_p/(1 - \epsilon_m) \). Using these parameters, Eq. (32-3) becomes

\[
R_m = \frac{K(1 - \epsilon_m)^2 S_m^2}{\epsilon_m}, \quad (32-4)
\]

where \( K = 2 \) for membranes with uniform cylindrical pores. For other membranes, Eq. (32-4) may still be used, but with the value of the constant \( K \) varying with the membrane