Chapter 10

Vector Quantization I: Structure and Performance

10.1 Introduction

Vector quantization (VQ) is a generalization of scalar quantization to the quantization of a vector, an ordered set of real numbers. The jump from one dimension to multiple dimensions is a major step and allows a wealth of new ideas, concepts, techniques, and applications to arise that often have no counterpart in the simple case of scalar quantization. While scalar quantization is used primarily for analog-to-digital conversion, VQ is used with sophisticated digital signal processing, where in most cases the input signal already has some form of digital representation and the desired output is a compressed version of the original signal. VQ is usually, but not exclusively, used for the purpose of data compression. Nevertheless, there are interesting parallels with scalar quantization and many of the structural models and analytical and design techniques used in VQ are natural generalizations of the scalar case.

A vector can be used to describe almost any type of pattern, such as a segment of a speech waveform or of an image, simply by forming a vector of samples from the waveform or image. Another example, of importance in speech processing, arises when a set of parameters (forming a vector) is used to represent the spectral envelope of a speech sound. Vector quantization can be viewed as a form of pattern recognition where an input pattern is “approximated” by one of a predetermined set of standard patterns, or in other language, the input pattern is matched with one of a stored set of
templates or codewords. Vector quantization can also be viewed as a front end to a variety of complicated signal processing tasks, including classification and linear transforming. In such applications VQ can be viewed as a complexity reducing technique because the reduction in bits can simplify the subsequent computations, sometimes permitting complicated digital signal processing to be replaced by simple table lookups. Thus VQ is far more than a formal generalization of scalar quantization. In the last few years it has become an important technique in speech recognition as well as in speech and image compression, and its importance and application are growing.

Our treatment of VQ in this book is motivated primarily by its value as a powerful technique for data compression. We hope, however, that the treatment presented here will provide a foundation for applications in pattern recognition as well.

The topics presented in this chapter and the next closely parallel those of Chapters 5 and 6 on scalar quantization. Many of the basic definitions and properties immediately generalize from scalars to vectors, while some do not generalize at all. These similarities and differences will be emphasized in the development.

We first present the basic definition of VQ and the structural properties that are independent of any statistical considerations or distortion measures. The structure and basic ideas for software or hardware implementation of VQ are considered for both the general case and the special case of uniform quantizers based on lattices. Basic complexity considerations are also presented. The presentation here assumes the reader has read or reviewed the basic material on scalar quantization presented in Chapters 5 and 6. We shall occasionally refer to “quantizers” or “quantization” implying the generality of VQ but without specifically attaching the modifier “vector.” Of course, scalar quantization is always a special case of VQ and all results can so be specialized. In Chapter 11 we continue the treatment of VQ by focusing on the optimality properties of vector quantizers and their implications for quantizer design. In Chapter 12 we consider special structures for VQ that often help to improve distortion-rate performance when a complexity constraint is placed on the encoding operation.

Basic Definitions

A vector quantizer $Q$ of dimension $k$ and size $N$ is a mapping from a vector (or a “point”) in $k$-dimensional Euclidean space, $\mathbb{R}^k$, into a finite set $C$ containing $N$ output or reproduction points, called code vectors or codewords. Thus,

$$Q : \mathbb{R}^k \to C,$$