"PORTFOLIO" PROBLEM, OPTIMAL INVESTMENT OF RESOURCES

1. INTRODUCTION

The previous examples illustrated competition and inspection processes in economical, social and ecological problems. Here the optimal investment of available resources is considered. Investment problems depend on the nature of resources to be invested. An important part of any investment problem is a proper definition of utility functions that determine the profit-to-risk relation. Here we consider an illustrative example how to invest some fixed capital in Certificates of Deposit (CD) and Stocks.

The portfolio problem is to maximize the average utility of a wealth. That is obtained by optimal distribution of available capital between different objects with uncertain parameters (Mockus et al., 1997). Denote by $x_i$ the part of the capital invested into an object $i$. The returned wealth is $y_i = c_i x_i$. Here $c_i = 1 + \alpha_i$ and $\alpha_i > 0$ is an interest rate. Denote by $p_i = 1 - q_i$ the reliability of investment. Here $q_i$ is the insolvency probability. $u(y)$ is the utility the wealth $y$. Denote by $U(x)$ the expected utility function. $U(x)$ depends on the capital distribution $x = (x_1, ..., x_n), \sum_i x_i = 1, x_i \geq 0$. If $y$ is continuous, the expected utility function

$$U(x) = \mathbb{E}u(y) = \int_0^\infty u(y)p(y)dy.$$  \hspace{1em} (13.1)
Here \( p(y) \) is probability density of wealth \( y \). If the wealth is discrete \( y = y^k, \ k = 1, \ldots, M \), the expected utility function

\[
U(x) = \sum_{k=1}^{M} u(y^k)p(y^k).
\]

(13.2)

Here \( M \) is the number of discrete values of wealth \( y^k \). \( p_x(y^k) \) is the probability that the wealth \( y^k \) will be returned, if the capital distribution is \( x \). We search for such capital distribution \( x \) which provides the greatest expected utility of the returned wealth:

\[
\max_x U(x),
\]

(13.3)

\[
\sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0.
\]

(13.4)

2. **EXPECTED UTILITY**

One may define probabilities \( p(y^j) \) of discrete values of wealth \( y^j, \ j = 1, 2, \ldots \) by exact expressions. For example,

\[
p(y^1) = p_1 \prod_{i \neq 1} q_i,
\]

\[
p(y^2) = p_2 \prod_{i \neq 2} q_i,
\]

\[\ldots\]

\[
p(y^n) = p_n \prod_{i \neq n} q_i,
\]

\[
p(y^{n+1}) = p_1 p_2 \prod_{i \neq 1, i \neq 2} q_i,
\]

\[
p(y^{n+2}) = p_1 p_3 \prod_{i \neq 1, i \neq 3} q_i
\]

(13.5)

Then from expression (13.5)

\[
U(x) = \sum_{k=1}^{M} u(y^k)p(y^k).
\]

(13.6)

Here \( M \) is the number of different values of wealth \( y \).