Chapter 24

Elementary linear calculations (vectors and matrices)

Section 24.1: introduction

The direct computing methods used in the preceding chapter for the trivariate normal distribution (sections 23.2 to 23.5) are not practical when there are more than two predictor variates. The most efficient manner of carrying out calculations is then based on vector and matrix algebra. Most variates considered in earlier chapters involved a single quantity, known as a scalar. A scalar quantity is generally denoted by an italic letter, such as

\[ X = \text{the stature (height) of a human adult} = 170 \text{ cm} \, . \]

Unlike scalars, vectors and matrices, which will be briefly described in this chapter, are ordered sets containing several elements which can be identified by their respective position. The elements are separated from each other either by commas or more simply by spaces. For instance, the following is a vector comprising three elements:

\[ [X, Y, Z] = [170 \text{ cm}, 50 \text{ cm}, 65 \text{ kg}] \, , \]

where \( X = \) the stature, \( Y = \) the shoulder width and \( Z = \) the body weight of a human adult. For the sake of conciseness and coherence, the vector as a whole is often denoted by a bold type letter and its successive elements by the same letter in italic type bearing inferior indices (order numbers, which are also called subscripts). The above vector may thus be written

\[ X = [X_1, X_2, X_3] = [170 \text{ cm} \quad 50 \text{ cm} \quad 65 \text{ kg}] \, , \]

where \( X_1, X_2 \) and \( X_3 \) stand for the variates \( X, Y \) and \( Z \) respectively. The elements of vectors and matrices are often scalars but may occasionally themselves be sets (see the case of subdivided matrices in section 24.14). Vector and matrix algebra is particularly efficient when several variates must be analyzed jointly because it allows these variates to be handled in an integrated manner. Moreover, algorithms (computational procedures) developed for vectors and matrices are extensible, which means that they may be used with any number of variates (provided that number is specified).

The reader may already know the geometrical meaning of the word vector: a straight-line segment which has a given length, a direction and a sense and which may be represented by an arrow. In algebra, a vector is an ordered set of two or several elements arranged in a row or in a column. Even though the algebraical and the geometrical meanings of the word vector are clearly different, both may often be considered as two aspects of a single entity. In this way, relationships may be perceived between mathematical entities which would otherwise appear to be unrelated.

In two-dimensional analytic geometry, for instance, an algebraic vector \([a, b]\) may be interpreted (figure 24.1.1) as the set of coordinates of a point, or else as the components
of a vector going from the origin \([0, 0]\) to the point \([a, b]\), or also as the direction numbers (numbers proportional to the direction cosines, see section 24.6) of a straight line passing through the origin \([0, 0]\) as well as through point \([a, b]\). Several of the words used in linear algebra have geometrical connotations which facilitate understanding.

\[
\begin{bmatrix}
X \\
y
\end{bmatrix} = \begin{bmatrix}
a \\
b
\end{bmatrix}
\]

**Figure 24.1.1**
Three possible geometrical interpretations of an algebraic vector \([a, b]\): the coordinates of a dot (left), the components of a vector (center), and the direction numbers of a straight line (right)

Section 24.2: row vectors and column vectors, transposition

In linear algebra, a vector is called a row vector if it is written horizontally but a column vector if it is written vertically, for these two kinds of vectors behave differently in some operations. However, a row vector may be transformed into a column vector or vice versa through an operation known as transposition, which may be indicated by placing a tilde (\(\sim\)) above the letter denoting the vector. In this textbook, both kinds of vectors may readily be recognized, for row vectors are written without a tilde and column vectors with a tilde (many authors place a transposition sign on row vectors rather than column vectors, but it may be more natural to treat row vectors as untransposed). Thus, transposing the row vector

\[
X = [X_1, X_2, X_3] = [170 \text{ cm} \ 50 \text{ cm} \ 65 \text{ kg}]
\]
yields the column vector

\[
\bar{X} = \begin{bmatrix}
X_1 \\
X_2 \\
X_3
\end{bmatrix} = \begin{bmatrix}
170 \text{ cm} \\
50 \text{ cm} \\
65 \text{ kg}
\end{bmatrix}
\]

Section 24.3: vector equality, addition and subtraction

The equality, the addition or the subtraction of two vectors implies that these two vectors are of the same kind (both are row vectors, or both are column vectors), and that they possess the same number of elements. Two vectors are equal if all corresponding elements are equal. For instance, if

\[
X = [X_1, X_2, X_3], \quad Y = [Y_1, Y_2, Y_3] \quad \text{and} \quad X = Y,
\]
this means that \(X_1 = Y_1\), \(X_2 = Y_2\) and \(X_3 = Y_3\).