SHAPES OF VESICLES AND CELLS UNDER FORCES EXERTED ON THE ENCLOSING MEMBRANE

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Shapes of vesicles and cells are governed and in simpler cases also determined by the mechanical properties of closed lamellar membranes. Lamellarity of biological membranes is manifested by the bilayer structure of phospholipid membranes and by the membrane skeletons and glycocalices positioned in parallel to the bilayer. When the layers forming a closed membrane are in contact but can slide in the lateral direction one past the other, the essential deformational modes are the area expansivity of the membrane neutral surface, the local membrane bending and the non-local membrane bending. Shapes of freely suspended vesicles or cells without internal structures can be predicted by assuming that they correspond to the minimum of the respective membrane elastic energy. When either external or internal, mainly skeleton-derived forces are exerted on their membranes, the shapes of vesicles and cells correspond to the minimum of the free energy of the system which in addition to the membrane elastic energy involves the potential energies of the forces. In this communication we shall first describe the contributions to the elastic energy of a closed bilayer and show the consequent shapes of freely suspended vesicles. Then two examples of how forces exerted on the vesicle membrane affect the vesicle shape will be presented. First we shall deal with the axial pulling force, and then by forces due to the external electric field.

We are considering flaccid vesicles, i.e. vesicles with their volume \( V \) smaller than the volume of the sphere \( (V_{\text{sph}}) \) of the same membrane area \( A \). The relative volume is defined as

\[
\nu = \frac{V}{V_{\text{sph}}} = \frac{6\pi^{1/2}}{\sqrt{V}} A^{1/2}
\]

As phospholipid membranes are not very expandable, the energy contribution due to membrane area expansivity does not essentially affect the shape of a flaccid vesicle, which is thus determined by the membrane local and non-local bending energies. Because of the property that the two leaflets of a bilayer can slide one past the other, the local bending energy is the sum of the bending energies of the two leaflets and can be expressed as

\[
\frac{1}{2} k_c \int (c_1 + c_2 - c_0)^2 \, dA
\]

where the bending modulus \( k_c \) is the sum of the bending moduli of the single leaflets, \( c_1 \) and \( c_2 \) are the principal curvatures, \( c_0 \) the spontaneous curvature, and integration is performed over the membrane area. The non-local bending energy can be expressed as

\[
\frac{1}{2} k_r \int (\Delta A - \Delta A_0)^2 / A \, dA
\]

where \( k_r \) is the non-
Figure 1. The correspondence between the relative difference of the areas of the membrane leaflets ($\Delta a$) and the equilibrium relative difference of the areas of the membrane leaflets ($\Delta a_0$). The values are calculated for stable axisymmetrical shapes at the relative volume $\nu = 0.85$, the ratio between the non-local and local bending moduli $k_r / k_c$ being equal to 3 and the spontaneous curvature $c_0$ being equal to zero, which is the value for a symmetrical bilayer. Boundary values for the intervals on $\Delta a$ and $\Delta a_0$ are given in the text. The representative examples of stable phospholipid vesicle shapes are also shown.

local bending modulus, $h$ the distance between the neutral surfaces of the membrane leaflets, $\Delta A$ the difference between the areas of the two leaflets, and $\Delta A_0$ the corresponding equilibrium difference. The area difference $\Delta A$ is equal to $h \left( c_1 + c_2 \right) dA$ and thus depends on the vesicle shape, whereas the equilibrium area difference $\Delta A_0$ depends on the composition of the leaflets and is constant when molecules do not transfer from one leaflet to the other and from or to the vesicle environment.

The axisymmetrical shapes which correspond to the minimum of the sum of the local and non-local bending energies at constant membrane area and vesicle volume can be determined by solving the corresponding variational problem which gives rise to a set of Euler-Lagrange equations. By neglecting thermal fluctuation, different shapes are obtained for different values of the relative volume $\nu$, the ratio of the two bending moduli $k_r / k_c$, the spontaneous curvature $c_0$, and the relative equilibrium difference between the areas of the two leaflets defined as $\Delta a_0 = \Delta A_0 / \Delta A_{\text{ph}}$, with $\Delta A_{\text{ph}} = 4h(\pi A)^{\nu - 1}$ being the difference between the areas of the two leaflets for a spherical vesicle. Actually, there are only three independent parameters because the same shapes (but with different energies) are obtained for different combinations of the parameters $c_0$ and $\Delta a_0$. The ratio $k_r / k_c$ for phospholipid membranes is in the range from 2 to 4. The analysis shows that for such values of this ratio the stable shapes can have, within a reasonable range of the values of $\Delta a_0$, only restricted values of the relative area difference $\Delta a$. Thus, at continuously increasing $\Delta a_0$, the values of $\Delta a$ exhibit several discontinuous changes which means that the corresponding shape transitions are discontinuous.

The detailed results for the relative volume $\nu = 0.85$ are presented in Fig. 1: in the range $-0.654 < \Delta a_0 < 0.553$ the realized shapes have the area difference $\Delta a$ in the range $0.664 < \Delta a < 0.712$ (represented by shape $a$), in the range $0.553 < \Delta a_0 < 0.913$ the realized shapes have $\Delta a \equiv 1.0289$ (shape $b$), in the range $0.913 < \Delta a_0 < 1.732$ the realized shapes have $\Delta a \equiv 1.0848$ (shape $c$), and in the range $1.732 < \Delta a_0 < 2.616$ the realized shapes are in the interval $1.269 < \Delta a < 1.301$ (represented by shape $d$). The shapes within the first and the last quoted intervals are characterized by narrow necks between the parent and daughter part of the vesicle and are schematically well represented by the limiting shapes $a$ and $d$, i.e. shapes composed of spheres connected by infinitesimal necks.