Chapter 9

SEQUENTIAL JOINT MAXIMIZATION
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Abstract: This paper describes a new method by which a competitive market economy may be represented as the optimal solution to a planning problem. The resulting sequential joint maximization (SJM) algorithm solves a sequence of "partial equilibrium relaxations" of the underlying general equilibrium model. The partial equilibrium submodels can be solved as nonlinear complementarity problems or as constrained nonlinear optimization problems in either a primal or dual form. This paper introduces the three SJM algorithms, evaluates performance and examines convergence theory. Computational tests demonstrate that SJM is not always as efficient as complementarity methods. A counterexample is presented which demonstrates that local convergence of SJM cannot be guaranteed. Although SJM may have neither the theoretical pedigree of a fixed-point algorithm nor the local convergence rate of a Newton algorithm, the usefulness of the SJM algorithm is demonstrated by the range of real models for which the procedure has been successfully applied. SJM benefits from the availability of robust, large-scale nonlinear programming codes. Consequently, for large scale equilibrium models with inequalities SJM may not be the "best" algorithm in theory, but it works extremely well in practice.

Introduction

There is a close connection between the allocation of a competitive market economy and the optimal solution to a representative agent's planning problem. This paper describes a new method by which this relationship can be exploited in computing equilibrium prices and quantities. This optimization—based algorithm can be applied to find equilibria for models in which consumers have heterogeneous preferences and where production sets may involve point-to-set mappings. Large-scale models may be solved through this technique. Extensions of the algorithm may be applied to models with tax distortions, price rigidities, monopolistic competition and arbitrary preferences. The present paper, however, only considers applications in which producers are perfectly competitive and consumer preferences are homothetic.
The algorithm is a Negishi or "joint maximization" procedure based on a sequence of convex nonlinear programming problems. Typically this sequence will converge to the equilibrium prices and quantities of a competitive market economy. This paper introduces and interprets the algorithm, provides some computational evidence and develops local convergence theory for a simple example.

The idea behind the SJM algorithm is found in the literature concerning conditions under which the demand function describing a set of heterogeneous consumers can be replaced by the demand function for a single agent. In many settings, this is known as the problem of exact aggregation. In neoclassical trade theory this relates to the "existence of community indifference curves". This literature investigates the conditions under which a country's offer curve may be represented as though it arose from a single optimizing agent. In trade theory, at least two such conditions have been identified. The first, due to Samuelson (1956), is that all consumers have identical, homothetic preferences. A second, less widely cited result is due to Eisenberg (1961) and Chipman (1974). This condition places restrictions on endowment vectors in addition to preferences. Eisenberg's result, as summarized by Chipman (1974), is:

"if each of m individuals has a fixed money income, and if they all have homogeneous utility functions (not necessarily identical, it must be emphasized), then their aggregate demand function is integrable, that is, it may be thought of as resulting from the maximization of some fictitious aggregate utility function, subject to total expenditure being equal to total income."

In the algorithm described in this paper, Eisenberg's aggregate demand function is embedded in an iterative procedure, which accounts for changes in the relative incomes of agents in the equilibrium system.

The aggregation approach developed by Eisenberg and subsequently interpreted in an economic context by Chipman is presented in a "primal" form. This approach begins with an explicit representation of consumer preferences in which the utility of an individual is specified in terms of quantities consumed of different goods. An analogous result is obtained using preferences characterized in a "dual" form (using the indirect utility function which maps prices and income into utility) which provides an interesting connection between the duality theory of linear programming and the duality of demand function theory. This formulation also renders the computational procedure applicable to models based on econometrically-estimated functions for which primal forms may not exist.

There is an interesting correspondence between the sequential joint maximization (SJM) algorithm and both Mathiesen's (1985) sequential linear complementarity (SLCP) algorithm and Goldsman and Harker's (1990) variational inequality (VI) algorithm for general equilibrium models. SJM can be regarded as a robust form of SLCP; one in which income and price adjustments are effectively decoupled. The SJM algorithm computes a sequence of "partial equilibrium relaxations" of the underlying general equilibrium model. Unlike SLCP (see Mathiesen, 1987), in SJM the choice of numéraire does not affect convergence.